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ABSTRACT

The purpose of the research project was to develop an applied or technical mathematics curriculum which would meet the needs of vocational-technical students at the community college level. The research project was divided into three distinct phases: Identifying the mathematical concepts requisite for job-entry competencies in each of the occupational areas, arranging these mathematical competencies into a structure of sequential units, and developing curriculum materials for each of these units. Staff members from 10 community colleges in Texas participated in the survey to help determine and validate job competencies for the occupational areas of diesel mechanics, auto mechanics, radio/TV repair, air conditioning, welding, machine shop, printing, drafting, and electronics. The specific project results included (1) a list of mathematical concepts requisite for entry-level competencies in each of the selected occupational areas, (2) a set of structured, sequential technical mathematics units designed to meet the needs of vocational-technical students in the selected occupational areas, (3) curriculum guides for each of the technical mathematics units, (4) self-instructional learning packets for each of the technical mathematics units, and (5) performance-based pre- and posttests for each of the technical mathematics units. This final report consists of four major components: Narrative report; the taxonomy (composite and individual) of competencies (appendix C); five exemplary minimodules (appendix D); and curriculum guides for each of the major divisions of competencies identified in the taxonomy (appendix E). Appendixes C and D constitute most of the document. (HD)

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Final Report

Project No. V0016VZ
Grant No. OEG-0-74-1706

Community College Technical Mathematics Project

Samuel L. Self

Texas A&M University
College of Education

College Station, Texas 77843

December 1975

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ABSTRACT

Community College Technical Mathematics Project (December 1975)
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The purpose of this research project was to develop an applied or technical mathematics curriculum which meets the needs of vocational-technical students. The major objectives accomplished during the research project included:

1. Identifying the mathematical concepts requisite for entry-level competencies to previously selected vocational-technical occupations;
2. Developing the identified mathematical concepts into an organized taxonomy of concepts for vocational-technical students; and
3. Developing curriculum materials and self-instructional learning packect (including pre- and post-tests) for five sequential concepts.

The research project was divided into three distinct phases: (1) identify the mathematical concepts requisite for job-entry competencies in each of the occupational area; (2) arrange these mathematical competencies into a structure of sequential units; and (3) develop curriculum materials for each of these units.

The specific results of this research project include: (1) a list of mathematical concepts requisite for entry-level competencies in each of the selected occupational area; (2) a set of structured, sequential technical mathematics units designed to meet the needs of vocational-technical students in the selected occupational areas; (3) curriculum guides for each of the technical mathematics units; (4) self-instructional learning packets for each of the technical mathematics units; (5) and performance-based pre- and post-tests for each of the technical mathematics units.

Preface

The genesis for the "Community College Technical Mathematics Study" evolved from a hypothesis developed by Dr. Bery McKinnerney, now the Associate Dean of Occupational Education at the El Paso Community College, and substaniated by administrators of technical programs at the community college level, that the current mathematics program in most community college settings was not meeting the needs of the vocational-technical students. This hypothesis was translated into a proposal for submission to the ultimate funding by the United State Office of Education under Part C of P. L. 90-576.

The final report, developed by the project staff, consists of four major components: the narrative report; the taxonomy of competencies (Appendix C); Five exemplary mini modules (Appendix D); and curriculum guides for each of the major divisions of competencies identified in the taxonomy (Appendix E).

The development of the taxonomy of competencies was the major activity associated with this study and hopefully can be used by other curriculum developers in developing curriculum to meet the needs of vocational-technical students. The mini modules developed in this project are only examples. Utilizing the taxonomy, other curriculum developers could elect to utilize other curriculum models.

The project staff is most appreciative of the support and cooperation received from the ten participating institutions as identified in Appendix A. As project director I would also like to acknowledge the contribution of each member of the project staff. These individuals are also identified in Appendix A. We trust that the materials presented in this report will be of assistance in overcoming the problem reflected in the hypothesis that was the genesis for this study.

Donald L. Clark
Associate Dean for Research
College of Education
Texas A&M University
December, 1975

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Chapter I - Introduction

Background of the Study

According to a 1972 report from the U.S. Department of Labor, the nation's work force will soon require 20% or less with a bachelor's or higher degree, while approximately 25% will require technical skills and approximately 55% will require vocational skills. In contrast to these statistics, approximately 75% of the educational experiences in the secondary schools of Texas and the nation are designed to meet the entrance requirements of colleges and universities offering a baccalaureate or higher degree. This implies that the entering students in a post-secondary institution should have completed at least two years of high school algebra and, in some instances, a course in trigonometry.

According to Paul N. Thompson (1971, p. iv), "Under the philosophy of the open-door colleges, technical and supportive mathematics courses are geared for the student who has successfully completed at least three years of high school mathematics." However, under the open-door policy, the entering students may have above average, average, or below average abilities. Thompson (1971, p. iv) also stated that "...the person who enters a junior or community college to obtain an associate degree in a skill field may be an 'average' student who has met with limited success in high school mathematics or has had no high school mathematics whatsoever".

Some of the characteristics of the entering technical student were identified in a recent five-year study conducted by Dr. Thomas J. McHale and Paul T. Witzke at the Milwaukee Area Technical College, (MATC). The students who participated in the study were required to have a high school diploma and have completed at least one year each of algebra and geometry in high school. The academic histories of these students were summarized by McHale and Witzke (A System of Instruction for Career Mathematics, 1972, p. 7) as follows:

1. 65% of 70% rank in the bottom half of their high school classes.
2. 50% to 55% have not taken more than the two required years of college preparatory mathematics.
3. In the two years of required high school math, the precent receiving a 'D', 'U', or 'not taking' increases from 35% to 40% in the second semester of geometry.
4. Only 35% to 40% have taken a high school physics course or claim some ability to use a slide rule.
5. Except for English, their ACT scores compare favorably with the norms for junior colleges.

Herbert F. R. Adams (1970, p. ix) stressed the importance of the applications of mathematical concepts when he stated, "There is no shortcut to learning mathematics, just as their is no shortcut to learning carpentry, welding, or electronics. The only way to become proficient in problem-solving is to solve problems".

Statement of the Problem

The philosophy of a Technical Mathematics program can be summarized in the following statements:

1. The content of mathematics courses should include only topics relevant for vocational-technical students; and
2. The vocational-technical students are not being trained to become mathematicians; therefore, the approach to mathematics for

these students should be intuitive and numerical and should not emphasize structure and proof.

What about the community colleges operating under "open-door" admission policies? Many entering students have not completed both algebra and geometry, and in many instances, neither. This points to the need for mathematics curriculum which rectifies this problem.

Existing technical mathematics curriculum and materials did not meet the needs of vocational-technical programs and students. In addition, the mathematical concepts requisite for entry-level competencies in most vocational-technical occupations were not identified. Hence, technical mathematics programs in vocational-technical programs in Texas Community Colleges needed improving and updating to provide a technical or applied mathematics program to hold improved vocational-technical programs.

Significance of the Program

The enrollment in post-secondary vocational programs is increasing at a rapid rate. Coupled with this fact is the substantial increase in community college enrollments. The current and future college enrollment trends seem to be toward community colleges, specifically vocational programs which provide students opportunities to obtain a practical education and thus, become productive members of society.

The trend toward an increased emphasis in vocational education cannot be described as just local or regional. All states are experiencing this increased student interest in vocational education. If vocational education expects to meet these students needs, an education which is practical and relevant to the goals of these students is required. A technical mathematics curriculum which involves practical, useful applications of theoretical concepts would not only benefit Texas Community Colleges, but also would benefit community colleges in other states.

Preliminary Study

The need for determining mathematics concepts requisite for entry-level competencies in vocational-technical occupations in Texas was verified by a preliminary investigation of 10 geographically selected Texas junior and community college programs (See Appendix A,). The study asked the following questions:

1. Is there a need for research to identify mathematical concepts requisite for success in vocational-technical programs?
2. Would the junior or community college participate in a research project designed to identify mathematical concepts requisite for success in vocational-technical programs?
3. What is the current status of mathematics programs for vocational-technical students in each of the participating junior or community colleges?
4. Is technical or applied mathematics useful to vocational-technical students?

A survey instrument to gather the data was designed in the form of an interview schedule. The researcher interviewed the Dean and/or Assistant Dean for Vocational-Technical Education in each participating institution. All results substantiated the need for research which would identify mathematical concepts requisite for success in vocational-technical programs. Each respondant exhibited an interest in participating in such a research project. The status of mathematical programs for vocational-technical students in participating institutions ranged from a completely traditional or academic mathematics approach to a completely technical or applied mathematics approach.

A majority of the respondants indicated that the mathematics programs for vocational-technical students should be the applied or technical mathematics approach to meet the students' needs. Several respondants indicated plans to convert their present traditional or academic mathematics programs into applied or technical mathematics programs for vocational-technical students. According to E.E. Walters (1973), Assistant Dean for Applied Sciences, Del Mar College, Corpus Christi, this action was prompted by a faculty survey to determine if academic or applied mathematics was needed. He stated that all responses favored an applied mathematics program. Walters indicated that students fail in the academic mathematics program not from inadequate knowledge of what they needed to know from it, but from the other concepts that are included that they don't need to know. Walters stated that "We are including too much math of the wrong kind in the academic math". He felt that a ladder concept should be developed for the applied or technical mathematics program.

Glarkson Groos (1973), Dean of Occupational Education, San Antonio College, responded to a question concerning usefulness of technical mathematics in the following manner: "We do know that we are doing a heck of a lot better with a technical mathematics course than we were with an academic math. In the content of the courses you will find the math is related to what the student is doing...". Frank Williams (1973), Dean for Occupational Education, St. Phillips College, stated that "...for those of us that have experienced the change from academic to technical mathematics, there is no question in our minds as to the need for technical mathematics which relates to the vocational-technical areas". Those deans and/or assistant deans for schools currently offering applied or technical mathematics indicated they developed their programs as they proceeded through the courses. They agreed that a study should be conducted to identify mathematical concepts requisite for job-entry competencies in vocational-technical occupations and which could be used to develop curriculum materials for instruction of applied or technical mathematics.

Truman Isbell (1973), Director of Vocational-Technical Education, Brazosport College, made the following statement concerning the question of transferability of applied or technical mathematics: "I'm beginning to quit worrying about whether it transfers or not. We need to teach theses students as if they are going to go out on a job. Teach them what they need to know and forget about transferability". Isbell felt the vocational-technical student should be taught those mathematical concepts which best prepare him to function on the job. He indicated that his technical mathematics program has been more successful in preparing students for job entry competencies than traditional or academic mathematics programs.

Out of ten junior or community colleges surveyed, eight offered mathematics for some of their vocational-technical programs. Of the eight junior or community colleges offering an academic mathematics program for vocational-technical students, several indicated a desire to convert their academic mathematics programs to technical or applied mathematics programs. The remaining junior colleges desired a partial conversion from academic mathematics to technical mathematics. A significant finding of the survey was the fact that most Allied Health Occupational programs did not require or desire formal mathematics courses. The mathematics needed for each of these occupations is included as an integral facet of the health related courses, such as Pharmacology.

The survey data indicated that all the junior or community colleges studied, offer and desire to continue the offering of technical mathematics courses. The most significant finding of the survey was the indication that junior and community colleges prefer technical or applied mathematics for the vocational-technical students and that technical mathematics curricula research should be conducted toward developing a mathematics curriculum relevant to the needs of the community college student.

The following conclusions were made from the survey of ten participating junior or community colleges.

1. A study should be conducted to identify mathematical concepts requisite for job-entry competencies for vocational-technical occupations.
2. The junior or community colleges contacted were willing to participate in such a study.
3. Applied or technical mathematics programs are more valuable to the vocational-technical students than academic or traditional mathematics programs.
4. Curriculum materials for applied or technical mathematics programs need to be developed in relation to specific vocational-technical program areas.

Chapter II - Methods

Goals & Objectives

This research project was designed to develop a base for an applied or technical mathematics curriculum which would meet the needs of vocational-technical students. The major objectives accomplished during the term of project include:

1. Identifying mathematical concepts requisite for entry-level competencies in nine selected vocational-technical occupations. (Welding, Radio & T.V. Repair, Machine Shop, Diesel Mechanics, Printing, Drafting and Design, Machine Shop, and Electronics.)
2. Developing the identified mathematical concepts into an organized heirarchy of units of competencies for vocational-technical students; and
3. Developing curriculum materials, self-instructional learning packets, and pre- and post-tests for selected sequential units.

Design and Procedures

Vocational-technical programs to be used in this research project were identified in the Texas Education Agency Table, "A challenge and a Choice". Realizing that consideration of all 117 occupational programs was beyond the scope of limited research, the project included only those Industrial Occupation programs and Technical Occupation programs offered by a minimum of 12 institutions listed in the "A Challenge and a Choice" table.

Those programs selected for use in the research project and the number of institutions offering such programs are given in TABLE 1.

TABLE 1
NUMBER OF INSTITUTIONS OFFERING SELECTED VOCATIONAL PROGRAMS

Occupational Program	Number of Institutions Offering Program
Air Conditioning and Refrigeration	26
Auto Mechanics	33
Diesel Mechanics	11
Printing	12
Radio and T.V. Repair	18
Electronics Technology	29
Drafting and Design Technology	37
Machine Shop	18
Welding	28

The research project was divided into three distinct phases:

1. Identifying mathematical concepts requisite for job-entry competencies in each of nine selected occupational areas;
2. Arranging these mathematical concepts into structures of sequential units; and
3. Developing curriculum materials representing these prescribed units.

Texas Community Colleges offering the various technologies were contacted and presented a brief outline of the project. The thirty-five texts used in teaching technical mathematics courses in Texas Community Colleges were identified (See Appendix B). The project staff used these texts as guides to construct a list of mathematical competencies. Each competency statement was examined to eliminate ambiguity and duplication. Based upon the competency listings, the staff constructed a research instrument in Likert (Guilford, 1964) scale form. The Likert instrument listed the mathematical competencies according to their skill levels including basic arithmetic, basic algebra, advanced algebra, trigonometry, and introductory calculus.

Within each level competencies were arranged in sequential order from simplest to most difficult. This arrangement provided a structured, hierarchical taxonomy of technical mathematics units that were of potential need to the technical-vocational student in the various occupational areas (See Appendix C).

Staff members visited 10 community college in August, 1974, and carefully explained the purpose of the project. The 10 community colleges included: Del Mar Junior College, Bee County College, San Jacinto College, Alvin Junior College, St. Phillip's College, San Antonio Junior College, El Paso Community College, Eastfield Community College, Lamar University (College of Technical Arts), and Brazosport College. Each community college agreed to participate in the project and offered their administrators, teachers and advisory board members as a jury to validate the competencies identified in the instrument. The institutions were selected to represent a uniform distribution of participants from all areas of Texas. The institutions were sincerely enthusiastic about the project, as was Joe Godsey, Director of Post-Secondary Occupational Education at the Texas Education Agency, the state funding agency for Texas Vocational Programs.

The staff consulted Dr. Beryl McKinnerney, Associate Dean of Occupational Education at El Paso Community College, to further refine the competency instrument. Staff members then visited El Paso Community College and conducted the first jury validation of competencies in September, 1974. The jury commented favorably on the completeness of the instrument and decided, under Dr. McKinnerney's supervision, to make helpful suggestions to further refine the instrument.

Staff members visited the College of Technical Arts at Lamar University and conducted the second jury validation of competencies. After further refinement, visitations were made to the remaining participant institutions, where the instrument was explained to the jury members and left with them to be completed at their convenience and later mailed to the project director.

Of the 105 jury members contacted, a total of 95 returns were received representing 87 useable responses. A profile of the jury members returning useable responses was made for each of the vocational areas. The profiles are summarized in TABLE 2, which indicates the mean number of years of teaching experience and/or industrial experience for jurors in each vocational area.

TABLE 2
PROFILE OF JURY MEMBERS

Vocational Area	Average Number of Years Teaching Experience	Average Number of Years Industrial Experience
Air Conditioning	6.9	18.4
Auto Mechanics	10	18.3
Diesel Mechanics	7.1	17.3
Drafting	9	7
Electronics	5.4	7.1
Machine Shop	10.3	13.1
Printing	7.5	8.7
Radio-T.V. Repair	5.2	14.8
Welding	5.6	12.6

All jury responses were tabulated onto a master sheet and a mean was calculated for each competency. The cut-off point for significance on the five-point Likert scale was chosen as a mean of four, reflecting agreement of jury members. A master taxonomy of the significant competencies was then composed reflecting the jury ratings. Additional taxonomies were composed for each occupational area. These were submitted to a jury for review and recommended changes and revisions were made.

Five related competencies were selected which were rated as necessary in a majority of the occupational areas. The five concepts dealing with fractions included: least common denominator, addition, subtraction, multiplication, and division. The researchers then developed self-instructional mini-module packets for each of these competencies (See Appendix D). Each packet included pre- and post-tests, and a set of audio tapes with problem workbooks and a programmed instruction unit with a set of exercises modeled after Flexer and Flexer (1971) and O'Malley and Polya (1963).

Following the initial development of the self-instructional packets, they were pilot tested at El Paso Community College, using Dr. Beryl McKinnerney as consultant, plus consultants Dr. Walter Stenning and Ms. Peggy Cooper-Harding of Texas A&M University. The modules were also pilot tested at Lamar University, College of Technical Arts, using Dr. Ken Shipper as consultant. At both schools, students were asked to complete a questionnaire which would give the researchers the reactions of the user concerning the usefulness of and the attitudes toward the individualized units.

Upon completion of the pilot tests, the materials were revised and a set of self-instructional packets was completed. These were pilot tested another time at El Paso Community College to determine the feasibility of further revision.

The last stage of the research project included the development of curriculum guides for the competencies rated as necessary for the various occupational areas. The curriculum guides were written following the sequencing indicated by the master taxonomy list and reflect observable student behaviors. Following the advice of curriculum experts, the curriculum guides do not reference any specific texts. The concensus of opinion was that this would lend greater flexibility to the implementation and continued use of guides (See Appendix E).

Chapter 3 - Results

The results of the project include:

1. A composite list of mathematical concepts requisite for entry-level competencies in the selected vocational areas;
2. A sequentially ordered taxonomy of mathematics concepts required to meet the needs of vocational-technical students in each of the ten selected vocational-technical areas with relative ratings by jurors for each occupational area;
3. Self-instructional learning packets for five related technical mathematics concepts;
4. Performance-based pre- and post-tests for the learning packets;
5. Curriculum guides for technical mathematics units in Basic Arithmetic, Basic Algebra, Advanced Algebra, and Trigonometry.

The composite and individual taxonomies were revised several times after the initial draft had been compiled. The revisions allowed the staff to update figures due to late arrival of some jurors' rating instruments. The revisions also led to a more readable format as suggested by consultants.

In order to check the effects of the self-instructional packets, each student participating in the pilot test was given a pre- and post-test over materials covered in the module he had worked. A t-test (Steel and Torrie, 1960) was used to test the null hypothesis $H_o: \mu_1 = \mu_2$ where μ_1=the mean of pre-test scores and μ_2= the mean of post-test scores. The test statistic used was $t = \bar{d}/s_{\bar{d}} = (\bar{x}_1 - \bar{x}_2)/s_{\bar{x}_1 - \bar{x}_2}$ since $d = \bar{x}_1 - \bar{x}_2$. The t value corresponding to Student's t, for 61 degrees of freedom showed a highly significant difference in pre- and post-test scores. With $\bar{x}_1=4.4$ and $\bar{x}_2=6.2$, results revealed that the two means differed at a probability level of $p<.001$.

The information on the student opinion questionnaire revealed that 79 percent of the students would use the modules, 14 percent might use them, and only 7 percent would not use the modules if they were available.

Students who found mathematics texts difficult to understand were interested in the possibility of widespread availability of cassette tapes. They were excited about being able to repeat parts of the tape if the need was felt. At El Paso Community College, the Mexican Americans were very enthusiastic over the tape aspects of the modules and suggested that translation into regionals Spanish might be beneficial for those minority groups.

The technical mathematics units and curriculum materials will be made available to the ten participating junior or community colleges plus other institutions both on a state and national basis upon request for implementation into their vocational-technical programs. The structured technical mathematics program, if implemented by community colleges offering vocational-technical programs, can serve as a firm basis for granting transferability of technical mathematics into similar four-year college or university programs. The findings of the project also have implications for curriculum development in other occupational areas.

A technical mathematics curriculum utilizing self-instructional models provides vocational students the opportunity to proceed at their own rate. The students can improve a weak mathematical background as well as develop mathematical competence to meet the pre-requisites of occupational programs.

Chapter 4 - Conclusions

This research project focused on identifying technical mathematics curriculum and guides specifically relevant to vocational-technical programs in Texas community colleges. The research will help solve current articulation problems, as well as improve existing vocational-technical programs.

When implemented, results of the research should benefit vocational-technical students, teachers, and employees by lowering the attrition rate of vocational-technical students in Texas community colleges. The taxonomies, curriculum guides and mini-modules can serve as a guide for developing full-size curriculum modules in selected vocational areas.

The structured, sequential technical mathematics curriculum will help in solving the articulation problems the vocational-technical student currently encounters in transfering between programs. The structured technical mathematics program, if implemented by all community colleges offering vocational-technical programs, can serve as a firm basis for granting transferability of technical mathematics into similar four-year college or university programs.

A technical mathematic program which utilizes self-instructional modules permits the vocational students to procede at their own rate. These modules allow students to improve a weak background in mathematics as well as develop mathematical competencies to meet the pre-requisites of their occupational programs.

The mathematical concept taxonomies and all materials that were pilot tested in this research project are now available to Texas community colleges. The taxonomy information can be used as a model to develop and test further taxonomies of mathematical competencies for any other occupational areas not covered in this research.

The results of the pilot tests showed that the self-instructional approach can be successful for the vocational-technical student. The success of the pilot tests and the favorable attitudes of both instructors and students toward the self-instructional modules suggest that further research into the mathematical competencies for vocational-technical programs in Texas community colleges should be undertaken in the form of more comprehensive curriculum modules and guides not only for the nine occupational areas used for this project but for other occupational areas as well.

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APPENDIX A

Participants in the Community College Technical Mathematics Project

Investigators and Consultants

Institutions

INDIVIDUALS PARTICIPATING IN THE COMMUNITY COLLEGE TECHNICAL MATHEMATICS PROJECT

Project Investigators

Samuel L. Self, Ph.D.
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K. E. Shipper, Ph.D.
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Lamar University

Walter F. Stenning, Ph.D.
Associate Professor
Texas A&M University

INSTITUTIONS PARTICIPATING IN THE COMMUNITY COLLEGE TECH. MATH. PROJECT

Institution Dean/Program Director	Participating Vocational Areas
ALVIN JUNIOR COLLEGE Donald E. Brown Associate Dean for Learning Resources	Electronics, Drafting & Design
BEE COUNTY COLLEGE Neal M. McBryde Dean of Occupational Education	Auto Mechanics, Radio & T.V. Repair, Electronics, Drafting & Design
BRAZOSPORT COLLEGE T. T. Isbell Director of Occupational Education	Electronics, Drafting & Design, Machine Shop, Welding
DEL MAR COLLEGE Lonnie J. Jones Assistant Dean Vocational & Adult Education	Air Conditioning & Repair, Diesel Mechanics, Radio & T.V. Repair, Welding
EASTFIELD COLLEGE Virginia Dobbs Associate Dean of Instruction	Printing, Electronics
EL PASO COMMUNITY COLLEGE Beryl R. McKinnerney Associate Dean of Occupational Education	Air Conditioning & Repair, Auto Mechanics, Drafting & Design
LAMAR UNIVERSITY Ken Shipper Dean College of Technical Arts	Air Conditioning & Repair, Diesel Mechanics, Machine Shop, Welding
SAN ANTONIO COLLEGE Clarkson Groos Director Occupational Education and Technology	Air Conditioning & Repair, Drafting & Design
SAN JACINTO COLLEGE Willard S. Walter Executive Coordinator School of Occupational-Technical Ed.	Auto Mechanics, Printing, Radio & T.V. Repair
ST. PHILLIPS COLLEGE Frank P. Williams Director Occupational Education & Technology	Air Conditioning & Repair, Auto Mechanics, Diesel Mechanics, Machine Shop, Welding
TARRANT COUNTY JUNIOR COLLEGE Henry E. Chitsey Director Program Development	Electronics, Machine Shop

APPENDIX B

Technical Mathematics Texts Used in Compiling Rating Instrument

Adams, H. F. R., Technical Mathematics, McGraw-Hill Company of Canada Limited, 1970.

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APPENDIX C

Taxonomies of Mathematics Concepts Needed for Entry-Level Employment in Selected Vocational Technical Areas

Composite Taxonomy

Individual Taxonomies

- Diesel Mechanics
- Auto Mechanics
- Radio/TV Repair
- Air Conditioning
- Welding
- Machine Shop
- Printing
- Drafting
- Electronics

Review of the current technical and academic mathematics textbooks (Appendix B) was used to compile a list of mathematical concepts considered to be relevant to a technical mathematics curriculum. These concepts were arranged on a survey instrument to be reviewed by a selected jury. The survey instrument was designed to use a Likert scale to identify those mathematical concepts which are necessary for job-entry level competencies in each of the following vocational areas:

1. Air Conditioning and Refrigeration
2. Auto Mechanics
3. Diesel Mechanics
4. Drafting and Design Technology
5. Electronics
6. Machine Shop
7. Printing
8. Radio and T.V. Repair
9. Welding

A 105 member jury was selected to complete the survey instrument and review the concepts identified as requisite for job-entry level competencies. Of the original jurors, 90% completed and returned the rating instrument to the research director. These jury members representing the vocational areas listed above were selected from ten participating junior colleges and technical schools. The group consisted of mathematics instructors, vocational area instructors, and advisory board members with industrial experience.

The principal investigator and staff members met with jurors to complete the survey instrument. The purpose of the personal contact between project personnel and jury representatives was chiefly to clarify any questions with regard to the rating instrument in order to obtain maximum input from jurors. The meetings also allowed staff members to get general feedback from jurors concerning just what types of results would be most helpful to vocational instructors and planners.

The lists in the pages that follow represent the results of the jury's ratings. The first list is a hierarchical arrangement of all competencies which were identified as being necessary for job-entry level competency in any of the nine vocational areas. The X's in the columns to the right of the competency list indicate which area(s) rated a particular competency as necessary. This composite list is designed to show which competencies are limited to only a few areas and which are common to most or all areas.

Following the composite list are individual competency lists for each of the vocational areas. Included with each list is the average rating for each competency by each type of juror (mathematics instructor, μM; vocational area instructor, μV; or industrial advisory board member, μA). A mean rating of 3.5 on the Likert scale (from 1 to 5) was selected as a cutoff point for the inclusion of a concept in the list. Additionally, there is a figure given which represents the percentage of jurors who rated the competency as a 4 or 5 (necessary or highly necessary) on the Likert scale. This vocational area breakdown will allow the reader to see what differences of opinion there were among the three types of jurors for each of the various competencies.

Composite Taxonomy

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
BASIC ARITHMETIC									
Real Numbers									
1. Basic operations with whole numbers:	X	X	X	X	X	X	X	X	X
2. Combined operations with whole numbers:	X	X	X	X	X	X	X	X	X
3. Definition of signed numbers:	X	X		X		X	X	X	X
4. Graphing signed numbers:		X				X	X	X	X
5. Basic operations with signed numbers:		X		X		X	X	X	X
6. Sets:						X			
7. One-to-one correspondence:							X		
8. Rounding off:		X	X	X	X	X	X	X	X
9. Prime numbers:	X	X	X	X	X	X	X	X	
10. Composite numbers:	X	X					X	X	
11. Greatest Common Diviser:	X	X		X	X		X	X	
12. Least Common Multiple:	X	X		X	X		X	X	
13. Absolute value:	X	X			X	X		X	X
Common Fractions									
1. Reading and writing	X	X		X	X	X	X	X	X
2. Reading a ruler:	X	X		X	X	X	X	X	X
3. Basic operation with fractions:	X	X		X	X	X	X	X	X

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
4. Equivalent fractions:	X	X			X	X	X	X	X
5. Converting mixed numbers to improper fractions:	X	X			X	X	X	X	
6. Basic operations with mixed numbers:	X	X			X	X	X	X	
7. Combined operations with mixed fractions:	X	X			X	X	X	X	
8. Proper and improper fractions	X	X			X	X	X	X	X
9. Complex numbers:		X			X	X			X
10. Combining like terms:	X	X			X	X		X	X
11. Reducing to lowest terms:	X	X			X	X	X	X	
12. Finding the Lowest Common Denominator:	X	X		X	X		X	X	X
13. Basic operation with common fractions and mixed fractions:	X	X			X	X	X	X	X
Decimal Fractions									
1. Reading and writing:	X	X		X	X	X	X	X	X
2. Rounding off:	X	X		X	X	X	X	X	X
3. Basic operations:	X	X		X	X	X	X	X	X
4. Table of decimal equivalents:	X		X	X	X	X	X	X	X
5. Fraction to decimal conversion:	X	X		X	X	X	X	X	X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
6. Decimal to fraction conversion:	X	X			X	X	X	X	X
Scientific Notation									
1. The concept of powers of 10:			X			X	X	X	X
2. Significant figures			X			X	X	X	X
3. Rounding off:			X			X	X	X	X
4. Basic operation using powers of 10:			X			X		X	X
5. Raising to powers using powers of 10:			X			X		X	X
6. Extracting square roots using powers of 10:			X			X		X	X
Squares and Square Roots									
1. Using the table of square roots:			X		X			X	X
2. Finding the square of a number:			X		X	X		X	X
3. Finding the square root of a number:			X		X	X		X	X
4. Finding the square root by the long division method:						X			
5. Basic operations of radicals:						X			X
6. The radical sign and index of roots:									X

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
7. The Pythagorean Theorem:			X		X			X	X
Percentage									
1. Definition:	X	X		X	X	X	X	X	X
2. Converting fractions to percents:	X	X		X	X	X	X	X	X
3. Converting percents to fractions:	X	X		X	X	X	X		
4. Converting decimals to percent:	X	X		X	X	X	X		X
5. Converting percents to decimals:	X	X		X		X	X		X
6. Finding the percentage given the base and rate:	X	X			X				X
7. Finding the rate given the percentage and base:	X	X			X				X
8. Finding the base given the rate and percentage:	X	X			X				X
9. Solving problems:	X	X			X	X	X	X	X
10. Price lists and discount:	X	X	X		X		X		
11. Markup:	X	X	X				X		
Ratio and Proportion									
1. Definition of ratio:	X	X			X	X	X	X	X
2. Reducation of ratio to lowest terms:	X	X			X	X	X	X	X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
3. Definition of proportion:	X	X			X	X	X	X	X
4. Finding the unknown term in a proportion:	X	X			X	X	X	X	X
5. Application of porportion:	X	X			X	X	X	X	X
6. The principle of levers:	X	X			X	X			
7. Principle of pulleys:	X	X			X	X			
8. Principle of gears:	X	X		X		X			
9. Principle of hydraulic press:	X	X				X			
10. The English system of weights and measures:	X	X		X	X	X	X	X	X
11. The metric of weights and measures:	X	X		X	X	X	X		X
12. The English-metric conversion:	X	X		X		X	X		X
13. The binary number system:			X						X
14. The computer number system:									X
15. Computing like a computer:									X
16. Binary to decimal conversion:			X						X
17. Decimal to binary conversion:			X						X
18. Basic operations of binary numbers:			X						X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
19. Multiplying and dividing measurements:	X	X	X	X	X		X	X	X
20. Table of equivalent measurements:	X	X		X	X		X	X	
Instruments of Measuring									
1. Linear scales:	X	X		X	X	X	X	X	X
2. Vernier calipers:	X					X	X	X	
3. Micrometers:	X	X				X		X	
4. The protractor:	X				X	X	X	X	
5. The planimeter:								X	
6. The guage number of wire:		X	X	X					X
7. The framing square:		X			X				
8. The tachometer:	X	X							
9. Logarithmic scales:								X	X
10. Angular scales:								X	
11. The circumference rule:								X	
Length									
1. Basic operations using length:	X	X		X	X	X	X	X	
2. Length as a measure of distance:	X	X		X	X	X		X	
3. Calculations of a rectangle (perimeter, width, length)		X		X	X	X		X	

Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
	OCCUPATIONAL AREA								
4. Calculations of a triangle and auxillary lines.					X	X	X	X	
5. Calculations of a circle:		X			X	X	X	X	
6. Finding the diameter of a circle when the area is given:		X			X	X		X	
7. The perimeter of the ellipse:					X			X	
8. The perimeter of polygons:								X	
Area									
1. As a measure of surface space:				X	X			X	
2. Calculations of four-sided figures (Parallelogram, trapezoid, square, and rectangle):				X	X			X	
3. Area calculations for a circle:		X		X	X	X		X	
4. Relationship of radius, diameter, circumference arcs, sectors, chords, and tangents:					X	X		X	
5. The properties of π					X	X		X	X
6. Area of a regular hexagon:					X				
7. Regular polygon:								X	

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
Volume									
1. Measures of volume:		X		X	X		X	X	
2. Volume determined by area:				X	X			X	
3. Volume of rectangular solids (cubes):					X			X	
4. Volume of a sphere:								X	
5. Segments and zones of spheres:								X	
6. Volume of a cylinder:		X						X	
7. Surface volume of solid and hollow cylinders:								X	
8. Heights of cylinders:								X	
9. Lateral surface of cylinders:								X	
10. Volume of compound figures:								X	
11. Volume of composite figures:								X	
Weight									
1. Measures of mass:		X			X	X	X		
2. Specific gravity:		X		X		X			
3. Density:		X		X		X			
4. Weight standards:		X		X	X	X	X		

Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
	OCCUPATIONAL AREA								
5. Weight of materials:		X		X	X		X	X	
6. Weight of castings from patterns:		X							
Time									
1. Units of time:	X		X			X	X	X	X
2. Distance in relations to time:	X		X			X		X	X
3. Frequency:	X		X	X		X	X		X
4. Period:	X		X			X	X		X
Money Calculations									
1. Price computation:		X	X	X	X		X	X	
2. Income computation:		X	X	X	X		X	X	
3. Interest computation:		X	X		X		X	X	
Scale Drawings									
1. Finding the drawing measure:					X	X		X	
2. Finding the actual dimensions of an object from the measured dimension of a diagram:					X	X		X	
3. Finding the area of a figure drawn to a certain scale:					X	X		X	

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
Slide Rule									
1. Reading the C and D scales:			X					X	
2. Slide rule operations (basic):			X				X		
3. Laws of exponents:			X						
4. Reciprocal, squares and square roots:			X						
5. Cubes and cube roots:			X						
6. Locating the decimal point:			X					X	
7. Combination problems:								X	
8. Extracting roots of numbers and raising powers:			X						
9. Cube roots:			X						
10. Multiplying and dividing numbers by π:			X						
11. The reciprocal scales:			X						
12. The tangent scale:			X						
13. The ARC tangent of a number:			X						
14. The sine scale:			X						
15. The ARC sine of a number:			X						
16. The ST scale:			X						
17. The cosine:			X						

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
18. The cotangent:			X						
19. Proportion scale:			X						
20. Finding the circumference and area of a circle:								X	
Geometry									
1. Straight line & circle:						X		X	X
2. Polar coordinates:									X
Geometrical Construction									
1. Bisect or find mid-point of a line:								X	
2. Bisect an angle:								X	
3. Bisect an arc:								X	
4. Erect a perpendicular to a line at a given point on the line:								X	
5. Erect a perpendicular to a line at the end of the line:								X	
6. Erect a perpendicular through a point not on the line:								X	
7. Draw a line through given points parallel to a given line:								X	
8. Divide a given line into a number of equal parts:								X	

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
9. Draw an angle equal to a given angle:								X	
10. Draw an equilateral triangle of given size:								X	
11. Draw a circle through three given points:								X	
12. Find the center of a circle or circle arc:								X	
13. Inscribe a square in a circle:								X	
14. Construct a square of given size:								X	
15. Inscribe a hexagon in a circle:								X	
16. Inscribe an equilateral triangle in a circle:								X	
17. Construct a hexagon with one of the sides on a given line:								X	
18. Draw a tangent to a circle at a point on the circumference:								X	
19. Draw a tangent to a circle through a point outside the circle:								X	
20. Draw a tangent to two circles of equal size:								X	

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
21. Draw an internal tangent to two circles of equal size:								X	
22. Draw an external tangent to two circles of unequal size:								X	
23. Draw an internal tangent to two unequal circles:								X	
BASIC ALGEBRA									
General Numbers									
1. Properties of real numbers:					X				X
2. Substitution of variables into algebraic expressions:						X		X	X
3. Translation of word expressions into algebraic expressions:					X	X		X	X
4. Order of operations:			X			X		X	X
5. Evaluating expressions containing variables:			X		X	X		X	X
6. Basic operations involving vatiables:			X			X		X	X
7. Constants and variables:						X		X	X
8. Positive and negative numbers:			X			X		X	X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
9. Powers and exponents:			X		X	X		X	X
10. Absolute value:								X	
11. Formulas:			X			X		X	X
12. Ratio and proportion:					X	X		X	X
Arithmetic Operations with Algebraic Expressions									
1. Basic operation with algebraic numbers:			X		X	X		X	X
2. Operations with signs of grouping:			X			X		X	X
3. The distributive law:			X			X			X
4. Equations involving parentheses:						X		X	X
5. Equations as symbolic sentences:					X	X		X	X
6. Removing parentheses used to indicate multiplication					X	X		X	X
7. First degree equations with signed numbers:					X	X		X	X
8. Adding and subtracting like terms:					X	X		X	X
9. Adding and subtracting polynomials			X		X	X		X	X

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
10. Multiplication and division of monomials and polynomials:					X	X			X
11. Inserting signs of grouping:						X		X	X
12. Rules of exponents:			X		X	X		X	X
13. The square of a binomial sum or difference:			X		X	X			X
14. The product of a binomial sum or difference:			X		X	X			X
15. Factoring the difference of two squares:			X						X
16. Factoring out of common factors:								X	X
17. Factoring polynomials:									X
18. Multiplication by a common factor:					X	X		X	X
19. Special products or expansions:					X				
20. Extracting square roots:			X		X	X			X
21. Factoring monomials from polynomials:									X
22. Symbols of grouping:					X	X		X	X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
23. Squaring binomials:									X
Algebraic Fractions									
1. Properties of algebraic fractions:						X			X
2. Equations involving fractions:						X		X	X
3. Basic operations with algebraic fractions:						X			X
4. The fractional exponents:									X
5. Finding the least common denominator:						X		X	
6. Reducing to lower terms:						X			
Inequalities									
1. Properties of inequalities:						X			
Equations									
1. First degree equations in one and two variables:						X		X	
2. Literal equation formulas:						X		X	
3. Equivalent, identical, and conditional equations:								X	

Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
	OCCUPATIONAL AREA								
4. Simplification of equations:						X		X	
5. Solution of equations in variables:						X		X	
6. Analysis of a formula:								X	
7. Transposition of terms:								X	
8. Transposition of factors:								X	
9. Translating problems into equations						X		X	
10. Operations with dimensional units:						X		X	
11. Solving systems of equations of two and three unknowns, algebraically:								X	
12. Dimensional units and conversion:						X		X	
13. Equations containing fractions and decimals:						X		X	
14. Degree of an Equation:								X	
15. Solving linear equations:						X		X	
16. Algebraic solutions to variables:								X	
17. Equations with unknowns in the denominators:								X	

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
18. Proportions:						X		X	
19. Proportionality:						X		X	
Linear Graphs									
1. Coordinate notation:						X		X	X
2. Plotting points on graphs:						X		X	X
3. Graphs of ordered pairs:						X		X	X
4. Displaying data with graphs:						X		X	X
5. Solving problems by means of graphs:						X		X	X
6. Graphs of linear equations:						X		X	X
7. Determining a linear function from two points:								X	
8. Graphing parallel lines:								X	
9. Graphing perpendicular lines:								X	
10. Graphing lines parallel to the axis:								X	
11. Graphing solutions of simultaneous equations:								X	
12. The slope of a line and Y intercept:								X	

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
13. The distance formula:								X	
14. Graphic solution of system of linear equations:								X	
15. Solution by addition and subtraction:								X	
16. Applications of simultaneous equations:								X	X
Quadratic Equations									
1. Solution of $ax^2 + bx + c = 0$:								X	X
2. Solution by the quadratic formula:								X	
Exponential Notation									
1. Laws of basic operations with exponents:			X					X	X
2. The law of zero exponents:								X	X
3. The law of exponents in raising to powers:			X					X	X
4. The law of exponents in extracting roots:			X					X	X
5. The law of exponents based on 10:			X					X	X
6. Power variations:			X						X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
7. j-operator:			X						X
8. The one operator:			X						X
9. Complex numbers:			X						X
10. Complex equations:			X						X
11. Squares of numbers:			X						X
Radicals									
1. Irrational numbers:									X
2. The square root of positive numbers:								X	X
Logarithmic Notation									
1. Difinition of a Logarithm:								X	
2. Properties of logarithms:									X
3. Characteristics:									X
4. Mantissa:									X
5. Determining logs from a table:								X	X
6. Finding powers using logarithms:									X
7. Finding roots using logarithms:									X
8. Interpolation using logarithms:									X
9. The logarithm of unity:								X	
10. Given log n, find n								X	X

Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
11. Given n, find log n:								X	X
12. Logarithms to the base e:									X
13. Time constant curves:									X
Sets and Logic									
1. Electric switch diagrams:									X
2. Venn diagrams:									X
3. Truth tables:									X
4. Tautologies:									X
Work and Power									
1. Work and power:	X	X							X
2. Horsepower of a steam engine:		X							
3. Horsepower of a gas engine:	X	X							
4. Brake horsepower:	X	X							
5. Electrical power-watt:	X	X		X					X
6. Mechanical efficiency of machines:	X	X		X					
Speed Ratio and Pulley Gears									
1. Gear trains:		X							
2. Idlers:		X							

Competency	OCCUPATIONAL AREA								
	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
3. Finding the number of teeth for a given speed ratio:		X							
4. Compound gearing:		X							
5. Worm and gear:		X							
6. Trains and spur, bevel, and worm gearing		X							
7. Pulley trains:		X							
Quadratic Equations and Formulas									
1. The meaning of quadratic equations:								X	
ADVANCED ALGEBRA									
Exponents									
1. The meaning of powers whose exponents are either "1" or "0":									X
2. The meaning of powers with negative whole-number exponents:									X
3. The reciprocals of powers and converting division to multiplication:									X
4. Converting multiplications of powers of divisions:									X
5. Equivalent forms of fractions containing powers:									X
6. Powers of fractions:									X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
7. Powers of powers:									X
8. Adding and subtracting letterterms which contain powers:									X
9. A review of powers and the laws of exponents:			X						X
10. The roots of multiplications and divisions:									X
11. The roots of powers:									X
12. Simplifying radicals by factoring out perfect powers:									X
13. Raising radicals to powers:									X
14. The meaning of powers with fraction exponents:									X
15. Fractional exponents and the laws of exponents and radicals powers and decimal exponents:									X
16. The meaning of roots and radicals:								X	X
Linear Graphs and Slope									
1. Identifying linear equations in "x" and "y":									X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
2. The intercepts of linear exponents:									X
3. Graphing linear equations by plotting intercepts:								X	X
4. A graphical representation of changes in "y" corresponding to increases in "x":								X	X
5. The slope of a line expressed as a signed number:								X	X
6. The slope of a line as a ratio of various pairs of changes in "x" and "y":									X
7. The slope-intercept form of linear equations:								X	
8. Writing linear equations in slope intercept form:								X	
9. Determining the slope of a line through two known points:									X
10. Slope and parallel lines:								X	
11. The equations of horizontal and vertical lines:								X	

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
Common and Natural Logarithms									
1. The power-of-ten form of numbers greater than 1:									X
2. Common logarithms of numbers greater than 1:									X
3. Common logarithms of numbers between 0 and 1:									X
4. Logarithmic scales:									X
5. Semi-log and log-log graphs:									X
6. Evaluations with formulas which contain "e":									X
7. Natural (base "e") logarithms:									X
8. Evaluating "Ln" formulas:									X
9. Evaluating "Ln" formulas when a conversion to power-of-"e" form is required:									X
Logarithms and Exponentials: Laws and Formulas									
1. The laws of common logarithms:									X

Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
2. The laws of natural logarithms:									X
TRIGONOMETRY									
Right Triangles and Trigonometric Ratios									
1. Relationship between θ and the coordinates of P:									X
2. Variations in right triangle problems:									X
3. Linear velocity:									X
4. Right triangles and angle-sum principle								X	X
5. Right triangles and pythagorean theorem:								X	X
6. Scaling method and right triangles:									X
7. The definition of the tangent ratio:								X	X
8. The tangent ratio of three specific angles:								X	
9. A partial table of tangent ratios:								X	
10. The sine ratio - a partial table:									X
11. The cosine ratio - a partial table:									X

Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
	OCCUPATIONAL AREA								
12. The complete table of trig ratios:								X	
13. Finding unknown sides in right triangles:								X	X
14. Finding unknown angles in right triangles:								X	X
Vectors and Phasors									
1. Definition of vectors and phasors:								X	X
2. Vectors on the rectangular coordinate system:								X	X
3. Basic operations with phasors:									X
4. The trig ratios of reference angles:								X	X
5. Finding the length and direction of a slanted vector:								X	X
6. Addition of vectors by the component method:									X
7. Finding the length and direction of a resultant:									X
8. Adding vectors which lie on the coordinate axes:									X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
9. Finding the vector-addend:									X
10. Adding more than two vectors:			X						X
11. Vector opposites, the zero-vector, and the state of equilibrium:									X
12. Equilibriants and the state of equilibrium:									X
Trigonometric Ratios of Standard-Position Angles									
1. Principle angles:			X					X	X
2. Standard-position angles between 0° and 360°:			X					X	X
3. Reference angles of standard-position angles between 0° and 360°:			X					X	X
4. Trigonometric ratios of angles between 90° and 360°:			X					X	
5. Finding the function of any angle:			X					X	X
6. Finding standard-position angles corresponding to known ratios:			X					X	X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
7. Finding the angle given the function:			X					X	
8. Trigonometric ratios of 0°, 90°, 180°, 270° and 360°:			X					X	X
9. The basic cycle of the sine-wave graph, graphing trig. functions:			X						X
10. Vector-addition and standard position angles:			X						X
11. Trigonometric ratios of angles greater than 360°:			X						
12. Related angles:									X
13. Other cycles of sine-wave graph:			X						X
14. Sine and tangent of very small angles:			X						
15. Functions of angles in quadrants I, II, III, and IV:			X					X	X
16. Definition of a "degree" in angles measurement:								X	X
17. Operations using minutes, degrees and seconds:			X					X	
18. Definition of radian in angle measurement:								X	X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
19. The relationship between degrees and radians:								X	X
20. Radians expressed in terms of "π":			X					X	X
21. Trig. ratios of angles expressed in radians:								X	X
Oblique Triangles									
1. The difinition of an oblique triangle:								X	
2. Solving oblique triangles by right-triangle methods:								X	
3. The law of sines:								X	
4. Using the law of sines to find unknown angles of oblique triangles:								X	
5. Limitations of using the law of sines:								X	
6. The law of cosines:								X	
7. Using the law of cosines to find unknown sides and angles of oblique triangles:								X	
8. Right triangles-law of sines and law of cosines:								X	
9. Strategies for solving triangles:								X	

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
10. The law of sines and obtuse angles:								X	
11. The law of cosines ang obtuse angles:								X	
Applied Geometric Problems									
1. Central angles and arcs of circles:								X	
2. Chords of circles:								X	
3. Basic problems involving chords and circles:								X	
4. Applied problems involving chords of circles:								X	
5. Sectors and segments of circles:								X	
6. Revolutions and rotational speed:								X	
7. Circumferential speed:								X	
8. Angular measures of rotational speed:								X	
9. Circumferential speed and angular velocity (in radians):								X	
10. Areas of triangles:								X	

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
Identities and Interpolation									
1. Definitions of the cosecant, and secant, and cotangents of an angle:								X	
2. Finding the cosecant, secant, and cotangent of an angle:								X	
3. The ratio identities:								X	
4. Pythagorean identities:								X	
5. Simple identities:								X	
6. Reciprocal identities:								X	
7. Verifying identities:								X	
8. Similar right triangles:								X	
Sine Wave Analysis									
1. Fundamental sine waves:									X
2. Non-fundamental sine waves:			X						X
3. Phase differences between non-fundamental and fundamental sine waves:			X						X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
4. The concepts of "lead" and "lag":			X						X
5. The broader interpretation of phase differences and the concepts of "lead" and "lag":			X						X
6. Sine wave harmonics:			X						X
7. Sketching fundamental sine waves:			X						X
8. Sketching non-fundamental sine waves:			X						X
9. Sketching harmonics of fundamental sine waves:			X						X
10. Sine waves with negative amplitudes:			X						X
Complex Numbers									
1. J-operator:			X						X
2. Real and imaginary numbers:			X						X
3. Complex numbers and their vector representation:			X						X
4. Vector-addition by means of complex numbers:			X						X
5. Vector-subtraction by means of complex numbers:			X						X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
6. The polar form of vectors which lies on an axis:			X						
7. Converting complex numbers from polar form to rectangular form:			X						
8. Graphs of complex numbers:			X						
Numerical Trigonometry (Applications)									
1. Measurement of angles:								X	
2. Mapping:								X	
3. Lattitudes and departures:								X	
4. Calculation of areas:								X	
5. Measurement of distances in a vertical plane:								X	
Periodic Functions									
1. Circular functions:									X
2. Graphing of $y=\sin\theta$:									X
3. Graph of $y=\cos\theta$:									X
4. Graphs of trig functions:									X
5. Trig functions; $f(x) = \sin(x)$; $g(x) = \cos(x)$:									X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
6. Amplitude:									X
7. Frequency:									X
8. Addition and sinusoidal waves:									X
THE D.C. CIRCUIT									
1. Difference of potential and electric current:		X	X	X					X
2. E.M.F. and energy considerations:		X	X	X					X
3. Ohm's law:		X	X	X					X
4. Voltage drops:		X	X	X					X
5. Energy and power dissipated in a circuit		X	X	X					X
6. Series and parallel circuits; equivalent resistance:		X	X	X					X
7. Direction of current:		X	X						X
8. Kirchhoff's rules:		X	X	X					X
9. Battery E.M.F. and terminal voltage -- internal resistance:		X	X						X
10. The Wheatstone bridge:		X	X	X					X
FUNDAMENTAL ELEMENTS IN AN A.C. CIRCUIT									
1. Capacitance:			X	X					X

	OCCUPATIONAL AREA								
Competency	Diesel Mechanics	Auto Mechanics	Radio/TV Repair	Air Conditioning	Welding	Machine Shop	Printing	Drafting	Electronics
2. Capacitors in series:			X	X					X
3. Capacitors in parallel:			X	X					X
4. Energy requires to charge a capacitor:			X	X					X
5. Time for charging and discharging a capacitor; the time constant:			X						X
6. Inductance:			X	X					X
7. Time constant of an L-R circuit:			X						X
8. Phase relation in A.C. circuits:			X	X					X
9. The series A.C. circuits phase diagram:			X	X					X
10. Reactance and impedance:			X	X					X
11. Resonance:			X						X
12. Power and power factor:			X	X					X

Diesel Mechanics

BASIC ARITHMETIC

Real Numbers

	μ V	μ M	μ A	μtot	%4-5
1. Basic Operations with Whole Numbers:	5	5	4	4.7	100%
2. Combined Operations with Whole Numbers:	5	5	4	4.7	100%
3. Definition of Signed Numbers:	3	5	4	4	67%
4. Basic Operations with Signed Numbers:	3	5	4	4	67%
5. Rounding Off:	4.8	5	5	4.9	100%
6. Prime Numbers:	3.8	5	4	4.3	67%
7. Composite Numbers:	3.8	5	4	4.3	83%
8. Greatest Common Diviser:	3.8	5	4	4.3	67%
9. Least Common Multiple:	4.3	5	4	4.4	83%
10. Absolute Value:	3	5	4	4	67%

Common Fractions

	μ V	μ M	μ A	μtot	%4-5
1. Reading and Writing:	5	5	5	5	100%
2. Reading a Ruler:	5	5	5	5	100%
3. Basic Operations with Fractions:	5	5	5	5	100%
4. Equivalent Fractions:	5	5	5	5	100%
5. Converting Mixed Numbers to Improper Fractions:	5	5	5	5	100%
6. Basic Operations with Mixed Numbers:	5	5	5	5	100%
7. Combined Operations with Mixed Numbers:	4	5	5	4.7	100%
8. Proper and Improper Fractions:	5	5	4	4.7	100%

	µ V	µ M	µ A	µtot	%4-5
9. Combining Like Terms:	3	5	5	4.3	83%
10. Reducing to Lowest Terms:	4	5	5	4.7	100%
11. Finding the Lowest Common Denominator:	5	5	4.5	4.8	100%
12. Basic Operations with Common Fractions and Mixed Fractions:	5	5	4.5	4.8	100%
Decimal Fractions					
1. Reading and Writing:	5	5	5	5	100%
2. Rounding Off:	5	5	5	5	100%
3. Basic Operations:	5	5	5	5	100%
4. Table of Decimal Equivalents:	5	5	5	5	100%
5. Fraction to Decimal Conversion:	5	5	5	5	100%
6. Decimal to Fraction Conversion:	5	5	5	5	100%
Percentage					
1. Definition:	5	5	5	5	100%
2. Converting Fractions to Percent:	5	5	5	5	100%
3. Converting Percents to Fractions:	5	5	5	5	100%
4. Converting Decimals to Percents:	5	5	5	5	100%
5. Converting Percents to Decimals:	5	5	5	5	100%
6. Finding the Percentage Given the Base and Rate:	4.8	5	5	4.9	100%
7. Finding the Rate Given the Percentage and Base:	4.8	5	4.5	4.8	100%
8. Finding the Base Given the Rate and Percentage:	4.8	5	4.5	4.8	100%

	μ V	μ M	μ A	μtot	%4-5
9. Solving Problems:	5	5	4	4.7	100%
10. Price List and Discount:	4.5	5	5	4.8	83%
11. Percentage Error and Averaging:	4.3	5	5	4.8	83%
12. Markup:	4.5	5	3	4.2	67%
Ratio and Proportion					
1. Definition of Ratio:	4.5	5	4	4.5	83%
2. Reduction of Ratio to Lowest Terms:	5	5	3	4.3	83%
3. Definition of Proportion:	5	5	3	4.3	83%
4. Finding the Unknown Term in a Proportion:	5	5	3	4.3	83%
5. Application of Proportion:	5	5	3	4.3	83%
6. Principle of Levers:	4.8	5	3	4.3	83%
7. Principle of Pulleys:	4.8	5	5	4.9	100%
8. Principle of Gears:	5	5	5	5	100%
9. Principle of Hydraulic Press:	5	5	5	5	100%
10. The English System of Weights and Measures:	5	5	5	5	100%
11. The Metric of Weights and Measures:	4.5	5	5	4.8	83%
12. English-Metric Conversion:	4	5	5	4.7	67%
13. Multiplying and Dividing Measurements:	4.3	5	4	4.4	83%
14. Tables of Equivalent Measurements:	4	5	5	4.7	67%
Instruments of Measuring					
1. Linear Scales:	4.3	5	5	4.8	83%

	μ V	μ M	μ A	μtot	%4-5
2. Vernier Calipers:	4.3	5	5	4.8	83%
3. Micrometers:	5	5	5	5	100%
4. The Protractor:	4.3	5	4	4.4	71%
5. The Tachometer:	5	5	5	5	100%
Length					
1. Basic Operations Using Length:	3	5	4	4	83%
2. Length as a Measure of Distance:	3	5	4	4	83%
Weight					
1. Measures of Mass:	4.5	5	2.5	4	72%
2. Specific Gravity:	4.8	5	3	4.3	86%
3. Weight Standards:	4	5	3	4	72%
4. Weight of Materials:	5	5	2.5	4.2	72%
Time					
1. Units of Time:	4.5	5	4	4.5	83%
2. Distance in Relation to Time:	3.8	5	4	4.3	67%
3. Frequency:	3.3	5	4	4.1	67%
Work and Power					
1. Work and Power:	4.5	5	5	4.8	83%
2. Horsepower of a Steam Engine:	4	4	4	4	72%
3. Horsepower of a Gas Engine:	5	4	5	4.7	100%
4. Brake Horsepower:	5	5	5	5	100%
5. Electrical Power -- Watt:	4.8	4	5	4.6	100%
6. Mechanical Efficiency of Machines:	5	5	5	5	100%

Auto Mechanics

	μ V	μ M	μ A	μtot	%4-5
BASIC ARITHMETIC					
Real Numbers					
1. Basic Operations with Whole Numbers:	5	5	5	5	100%
2. Combined Operations with Whole Numbers:	5	5	5	5	100%
3. Definition of Signed Numbers:	4.8	5	5	4.9	100%
4. Graphing Signed Numbers:	3.8	5	5	4.6	88%
5. Basic Operations with Signed Numbers:	4.5	5	5	4.8	100%
6. Rounding Off:	5	5	5	5	100%
7. Prime Numbers:	4.5	5	5	4.8	88%
8. Composite Numbers:	4	5	5	4.7	75%
9. Greatest Common Divisor:	5	5	5	5	100%
10. Least Common Multiple:	5	5	5	5	100%
11. Absolute Value:	4.3	4.5	5	4.6	88%
Common Fractions					
1. Reading and Writing:	5	5	5	5	100%
2. Reading a Ruler:	5	5	5	5	100%
3. Basic Operation with Fractions:	5	5	5	5	100%
4. Equivalent Fractions:	5	5	5	5	100%
5. Converting Mixed Numbers to Improper Fractions:	4.8	5	5	4.9	100%
6. Basic Operations with Mixed Numbers:	5	5	5	5	100%
7. Combined Operations with Mixed Numbers:	5	5	5	5	100%
8. Proper and Improper Fractions:	5	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
9. Complex Numbers:	4	5	5	4.7	88%
10. Combining Like Terms:	4.8	5	5	4.9	100%
11. Reducing to Lowest Terms:	4.8	5	4	4.6	100%
12. Finding the Lowest Common Denominator:	4.8	5	5	4.9	100%
13. Basic Operations with Common Fractions and Mixed Fractions:	4.8	5	5	4.9	100%
Decimal Fractions					
1. Reading and Writing:	5	5	2	4	88%
2. Rounding Off:	5	5	2	4	88%
3. Basic Operations:	5	5	2	4	88%
4. Fraction to Decimal Conversion:	5	5	2	4	88%
5. Decimal to Fraction Conversion:	5	5	2	4	88%
Percentage					
1. Definition:	4.8	5	3	4.3	88%
2. Converting Fractions to Percent:	4.8	5	3	4.3	88%
3. Converting Percents to Fractions:	4.8	5	3	4.3	88%
4. Converting Decimals to Percent:	4.8	5	3	4.3	88%
5. Converting Percents to Decimals:	4.8	5	3	4.3	88%
6. Finding the Percentage Given the Base and Rate:	4.5	5	3	4.3	88%
7. Finding the Rate Given the Percentage and Base:	4.5	5	3	4.3	88%
8. Finding the Base Given the Rate and Percentage:	4.5	5	3	4.3	88%

	μ V	μ M	μ A	μtot	%4-5
9. Solving Problems:	4.8	5	5	4.9	100%
10. Price Lists and Discount:	4.8	5	5	4.9	100%
11. Percentage Error and Averaging:	4.3	5	3	4.1	63%
12. Markup:	4.3	5	5	4.8	75%
Ratio and Proportion					
1. Definition of Ratio:	4.8	5	3	4.3	88%
2. Reduction of Ratio to Lowest Terms:	4.8	5	3	4.3	88%
3. Definition of Proportion:	4.8	5	3	4.3	88%
4. Finding the Unknown Term in a Proportion:	4.8	5	3	4.3	88%
5. Application of Proportion:	4.5	5	3	4.2	75%
6. The Principle of Levers:	4.8	5	3	4.3	88%
7. Principle of Pulleys:	4.8	5	3	4.3	88%
8. Principle of Gears:	5	5	3	4.3	88%
9. Principle of Hydraulic Press:	5	5	3	4.3	88%
10. The English System of Weights and Measures:	5	4.5	3	4.2	88%
11. The Metric of Weights and Measures:	5	4.5	3	4.2	88%
12. English-Metric Conversion:	5	4.5	3	4.2	88%
13. Multiplying and Dividing Measurements:	5	5	3	4.3	100%
14. Tables of Equivalent Measurements:	5	5	3	4.3	100%

	μ V	μ M	μ A	μtot	%4-5
Instruments of Measuring					
1. Linear Scales:	4	5	4	4.3	75%
2. Vernier Calipers:	4.5	5	3	4.2	75%
3. Micrometers:	5	5	5	5	100%
4. The Protractor:	4.8	5	3	4.3	88%
5. The Gauge Number of Wire:	5	5	5	5	100%
6. The Framing Square:	3.5	5	4	4.2	50%
7. The Tachometer:	5	5	5	5	100%
Length					
1. Basic Operations Using Length:	4	5	5	4.7	88%
2. Length as a Measure of Distance:	3	5	5	4.3	88%
3. Calculations of a Rectangle (Perimeter, Width, Length):	3.9	5	3	4	63%
4. Calculations of a Circle:	4.3	5	3	4.1	75%
5. Finding the Diameter of a Circle when the Area is Given:	4.5	5	3	4.2	75%
Area					
1. As a Measure of Surface Space:	4	5	3	4	63%
2. Calculations of Four Sided Figures (Parallelogram, Trapezoid, Square, and Rectangle):	4	5	3	4	63%
3. Area Calculations for a Circle:	4.8	5	3	4.3	88%
Volume					
1. Measures of Volume:	4.8	5	4	4.6	88%

	μ V	μ M	μ A	μtot	%4-5
2. Volume of a Cylinder:	4.8	5	3	4.3	88%
Weight					
1. Measures of Mass:	4.5	5	2.5	4	88%
2. Specific Gravity:	4.8	5	3	4.3	88%
3. Density:	4.8	5	2.5	4.1	88%
4. Weight Standards:	4.5	5	2.5	4	75%
5. Weights of Materials:	4.8	5	2.5	4.1	88%
Time					
1. Units of Time:	5	5	3	4.3	88%
2. Distance in Relation to Time:	5	5	2	4	75%
Money Calculations					
1. Price Computations:	4.5	5	2.5	4	75%
2. Income Computations:	4.5	5	2.5	4	75%
3. Interest Computation:	4.5	5	2.5	4	75%
BASIC ALGEBRA					
Work and Power					
1. Work and Power:	4.5	5	5	4.8	88%
2. Horsepower of a Steam Engine:	4	4	5	4.3	75%
3. Horsepower of a Gas Engine:	5	4	5	4.7	88%
4. Brake Horsepower:	5	5	5	5	100%
5. Electrical Power-Watt:	4.8	5	5	4.9	100%
6. Mechanical Efficiency of Machines:	5	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
Speed Ratio and Pulley Gears					
1. Gear Trains:	4.8	3	5	4.3	75%
2. Idlers:	4.8	3	5	4.3	75%
3. Finding the Number of Teeth for a Given Speed Ratio:	5	3	4	4	75%
4. Compound Gearing:	5	3.5	3.5	4	75%
5. Worm and Gear:	5	3	4.5	4.2	88%
6. Trains and Spur, Bevel and Worm Gearing:	5	3	4.5	4.2	88%
7. Pulley Trains:	5	3	4.5	4.2	88%
THE D.C. CIRCUIT					
1. Difference of Potential and Electric Current:	5	4.5	5	4.8	100%
2. E.M.F. and Energy Considerations:	5	4.5	5	4.8	100%
3. Ohm's Law:	5	4.5	5	4.8	100%
4. Voltage Drops:	5	4.5	5	4.8	100%
5. Energy and Power Dissipated in a Circuit:	5	4.5	5	4.8	100%
6. Series and Parallel Circuits; Equivalent Resistance:	5	4.5	5	4.8	100%
7. Direction of Current:	5	4.5	5	4.8	100%
8. Kirchhoff's Rules:	3.5	4.5	5	4.3	83%
9. Battery E.M.F. and Terminal Voltage -- Internal Resistance:	5	4.5	5	4.8	100%
10. The Wheatstone Bridge:	3	4.5	5	4.2	66%

Radio and T.V. Repair

	μ V	μ M	μ A	μtot	%4-5
BASIC ARITHMETIC					
Real Numbers					
1. Basic Operations with Whole Numbers:	5	5	3	4.3	83%
2. Combined Operations with Whole Numbers:	4.7	5	3	4.2	83%
Decimal Fractions					
1. Table of Decimal Equivalents:	4.3	5	3	4.1	67%
Scientific Notation					
1. The Concept of Powers of 10:	5	4	5	4.7	100%
2. Significant Figures:	5	4	5	4.7	100%
3. Rounding Off:	5	4	5	4.7	100%
4. Basic Operations Using Powers of 10:	5	4	5	4.7	100%
5. Raising to Powers Using Powers of 10:	5	4	5	4.7	100%
6. Extracting Square Roots Using Powers of 10:	5	4	5	4.7	100%
Squares and Square Roots					
1. Using the Table of Square Roots:	4.3	5	5	4.8	83%
2. Finding the Square of a Number:	4.7	5	5	4.9	100%
3. Finding the Square Root of a Number:	4.7	5	5	4.9	100%
4. The Pythagorean Theorem:	4	5	5	4.7	83%
Percentage					
1. Definition of Percent:	4	3	5	4	60%

	μ V	μ M	μ A	μtot	%4-5
2. Price Lists and Discount:	4	3	5	4	60%
3. Percentage Error and Averaging:	4.3	3	5	4.1	60%
4. Markup:	4.3	3	5	4.1	60%
Ratio and Proportion					
1. The Binary Number System:	4.3	3	5	4.1	67%
2. Binary to Decimal Conversion:	4.3	3	5	4.1	67%
3. Decimal to Binary Conversion:	4.3	3	5	4.1	67%
4. Basic Operations of Binary Numbers:	4.3	3	5	4.1	83%
Instruments of Measuring					
1. The Gauge Number of Wire:	4.7	3	5	4.1	83%
Time					
1. Units of Time:	4.7	5	5	4.9	100%
2. Distance in Relation to Time:	4.7	5	5	4.9	100%
3. Frequency:	5	4	5	4.7	100%
4. Period:	5	4	5	4.7	100%
Money Calculations					
1. Price Computation:	4	4	5	4.3	83%
2. Income Computation:	4	4	5	4.3	83%
3. Interest Computation:	4	4	5	4.3	83%
Slide Rule					
1. Reading the C and D Scales:	4	4	5	4.3	83%

	μ V	μ M	μ A	μtot	%4-5
2. Slide Rule Operations (Basic):	4	4	5	4.3	83%
3. Laws of Exponents:	5	4	5	4.7	100%
4. Reciprocal, Squares and Square Roots:	4.5	4	5	4.8	83%
5. Cubes and Cube Roots:	3.7	4	5	4.2	83%
6. Locating the Decimal Point:	5	5	5	5	100%
7. Combination Problems:	3.7	4	5	4.2	83%
8. Extracting Roots of Numbers and Raising Powers:	4	4	5	4.3	83%
9. Cube Roots:	3.7	4	5	4.2	67%
10. Multiplying and Dividing Numbers by π:	3.7	4	5	4.2	83%
11. The Reciprocal Scale:	3.3	4	5	4.1	83%
12. The Tangent Scale:	4	3	5	4	67%
13. The ARC Tangent of a Number:	4	3	5	4	67%
14. The Sine Scale:	4	3	5	4	67%
15. The ARC Sine of a Number:	4	3	5	4	67%
16. The ST Scale:	4	3	5	4	67%
17. The Cosine:	4	3	5	4	67%
18. The Cotangent:	4.5	3	5	4.2	67%
19. Proportion Scale:	3.3	5	4	4.1	83%

BASIC ALGEBRA

General Numbers

1. Order of Operations:	4	5	3	4	83%
2. Evaluating Expressions Containing Variables:	4	5	3	4	83%

	μ V	μ M	μ A	μtot	%4-5
3. Basic Operations Involving Variables:	4	5	3	4	83%
4. Positive and Negative Numbers:	4.7	5	5	4.9	100%
5. Powers and Exponents:	4.7	5	5	4.9	100%
6. Formulas:	4.7	5	5	4.9	100%
Arithmetic Operations with Algebraic Expressions					
1. Basic Operations with Algebraic Numbers:	4	5	4.5	4.5	83%
2. Operations with Signs of Grouping:	4	5	4.5	4.5	83%
3. The Distributive Law:	3.3	5	4.5	4.3	67%
4. Adding and Subtracting Polynomials:	3	5	5	4.3	50%
5. Rules of Exponents:	5	5	5	5	100%
6. The Square of a Binomial Sum or Difference:	3.7	5	5	4.6	67%
7. The Product of a Binomial Sum or Difference:	3.7	5	5	4.6	67%
8. Factoring the Difference of Two Squares:	2.7	5	5	4.2	67%
9. Extracting Square Roots:	3.3	4	5	4.1	83%
Exponential Notation					
1. Laws of Basic Operations with Exponents:	4.3	5	5	4.8	100%
2. The Law of Exponents in Raising to Powers:	4	5	5	4.7	83%
3. The Law of Exponents in Extracting Roots:	4	5	5	4.7	83%
4. The Law of Exponents Based on 10:	4	5	5	4.7	83%
5. Power Variations:	3.7	5	5	4.6	67%

	μ V	μ M	μ A	μtot	%4-5
6. j-Operator:	4	4	5	4.3	83%
7. The One Operator:	3.3	4	5	4.1	67%
8. Complex Numbers:	3.7	4	5	4.2	67%
9. Complex Equations:	3.3	4	5	4.1	67%
10. Squares of Numbers:	3.7	5	5	4.2	67%
TRIGONOMETRY					
Vectors & Phasors					
1. Adding More than Two Vectors:	3.7	5	5	4.6	83%
Trigonometric Ratios of Standard-Position Angles					
1. Principle Angles:	3	5	5	4.3	67%
2. Standard-Position Angles Between 0° and 360°:	4.3	5	5	4.8	83%
3. Reference Angles of Standard-Position Angles Between 0° and 360°:	4	5	5	4.7	83%
4. Trigonometric Ratios of Angles Between 90° and 360°:	3	5	5	4.3	67%
5. Finding the Function of any Angle:	3	5	5	4.3	67%
6. Finding Standard-Position Angles Corresponding to Known Ratios:	3	5	5	4.3	67%
7. Finding the Angle Given the Function:	3	5	5	4.3	67%
8. Trigonometric Ratios of 0°, 90°, 180°, 270°, and 360°:	3.3	5	5	4.4	67%
9. The Basic Cycle of the Sine-Wave Graph, Graphing Trig. Functions:	3.7	5	5	4.6	83%
10. Vector-Addition and Standard Position Angles:	3.3	4	5	4.1	67%

	µ V	µ M	µ A	µtot	%4-5
11. Trigonometric Ratios of Angles Greater Than 360°:	3.3	4	5	4.1	67%
12. Other Cycles of the Sine-Wave Graph:	4	4	5	4.3	83%
13. Sine and Tangent of Very Small Angles:	3	5	5	4.3	67%
14. Functions of Angles in Quadrants I, II, III, and IV:	3.3	5	5	4.4	67%
15. Operations Using Minutes, Degrees, and Seconds:	3	5	5	4.3	67%
16. Radians Expressed in Terms of "π":	4.3	5	5	4.8	100%
Sine Wave Analysis					
1. Non-Fundamental Sine Waves:	4.3	4	5	4.4	67%
2. Phase Differences Between Non-Fundamental and Fundamental Waves:	4.7	4	5	4.6	100%
3. The Concepts of "Lead" and "Lag":	4.7	4	5	4.6	100%
4. The Broader Interpretation of Phase Differences and the Concepts of "Lead" and "Lag":	4.3	3	5	4.1	67%
5. Sine Wave Harmonics:	4	3	5	4	67%
6. Sketching Fundamental Sine Waves:	4.3	4	5	4.4	83%
7. Sketching Non-Fundamental Sine Waves:	4	3	5	4	67%
8. Sketching Harmonics of Fundamental Sine Waves:	4	3	5	4	67%
9. Sine Waves with Negative Amplitudes:	4.3	4	5	4.4	83%
10. Cosine Waves:	4	4	5	4.3	83%

	μ V	μ M	μ A	μtot	%4-5
Complex Numbers					
1. j-Operator:	3.7	4	5	4.2	67%
2. Real and Imaginary Numbers:	3.7	4	5	4.2	67%
3. Complex Numbers and Their Vector Representation:	3.7	4	5	4.2	67%
4. Vector-Addition by Means of Complex Numbers:	3.7	4	5	4.2	67%
5. Vector-Subtraction by Means of Complex Numbers:	3.7	4	5	4.2	67%
6. The Polar Form of Vectors Which Lie on an Axis:	3.7	4	5	4.2	67%
7. Converting Complex Numbers From Polar Form to Rectangular Form:	3.7	4	5	4.2	67%
8. Graphs of Complex Numbers:	3.3	4	5	4.1	67%
THE D.C. CIRUCIT					
1. Difference of Potential and Electric Currents:	5	5	5	5	100%
2. E.M.F. and Energy Considerations:	5	5	5	5	100%
3. Ohm's Law:	5	5	5	5	100%
4. Voltage Drops:	5	5	5	5	100%
5. Energy and Powers Dissipated in a Circuit:	5	5	5	5	100%
6. Series and Parallel Circuits; Equivalent Resistance:	5	5	5	5	100%
7. Direction of Current:	5	5	5	5	100%
8. Kirchhoff's Rules:	4.3	5	5	4.8	83%
9. Battery E.M.F. and Terminal Voltage -- Internal Resistance:	5	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
10. The Wheatstone Bridge:	4.3	5	5	4.8	83%
FUNDAMENTAL ELEMENTS IN AN A.C. CIRCUIT					
1. Capacitance:	5	5	5	5	100%
2. Capacitors in Series:	5	5	5	5	100%
3. Capacitors in Parallel:	5	5	5	5	100%
4. Energy Required to Charge a Capacitor:	4.7	5	5	4.9	100%
5. Time for Charging and Discharging a Capacitor; the Timer Constant:	4.3	5	5	4.8	83%
6. Inductance:	5	5	5	5	100%
7. Time Constant of an L-R Circuit:	4.3	5	5	4.8	83%
8. Phase Relation in A.C. Circuits:	4.7	5	5	4.9	100%
9. The Series A.C. Circuits; Phase Diagram:	4.7	5	5	4.9	100%
10. Reactance and Impedance:	4.7	5	5	4.9	100%
11. Resonance:	5	5	5	5	100%
12. Power and Power Factor:	4.7	5	5	4.9	100%

Air Conditioning

	μ V	μ M	μ A	μtot	%4-5
BASIC ARITHMETIC					
Real Numbers					
1. Basic Operations with Whole Numbers:	5	5	5	5	100%
2. Combined Operations with Whole Numbers:	5	5	5	5	100%
3. Definition of Signed Numbers:	5	5	5	5	100%
4. Basic Operations with Signed Numbers:	3.6	4.3	5	4.3	70%
5. Rounding Off:	3.8	5	5	4.6	80%
6. Prime Numbers:	3.2	5	5	4.4	80%
7. Greatest Common Divisor:	3.2	4.6	5	4.3	70%
8. Least Common Multiple:	3	4	5	4	60%
Common Fractions					
1. Reading and Writing:	5	4.3	5	4.8	90%
2. Reading a Ruler:	5	5	5	5	100%
3. Basic Operations with Fractions:	5	4.3	5	4.8	90%
4. Equivalent Fractions:	5	4.3	5	4.8	90%
5. Converting Mixed Numbers to Improper Fractions:	3.2	4	5	4.1	70%
6. Basic Operations with Mixed Numbers:	4	3.5	5	4.2	70%
7. Combined Operations with Mixed Fractions:	4	3.3	5	4.1	70%
8. Proper and Improper Fractions:	4	3.3	5	4.1	70%
9. Combining Like Terms:	4	3.3	5	4.1	70%
10. Reducing to Lowest Terms:	3.8	3.6	5	4.1	60%

	μ V	μ M	μ A	μtot	%4-5
11. Finding the Lowest Common Denominator:	3.4	4	5	4.1	70%
12. Basic Operations with Common Fractions and Mixed Fractions:	4	4	5	4.3	80%
Decimal Fractions					
1. Reading and Writing:	5	5	5	5	100%
2. Rounding Off:	3.8	5	5	4.6	80%
3. Basic Operations:	4.2	5	5	4.7	90%
4. Table of Decimal Equivalents:	3.6	3.6	5	4.1	70%
5. Fraction to Decimal Conversion:	4	3.6	5	4.2	80%
6. Decimal to Fraction Conversion:	4	3.6	5	4.2	80%
Percentage					
1. Definition:	4.2	3	5	4.1	70%
2. Converting Fractions to Percent:	4.2	3	5	4.1	70%
3. Converting Decimals to Percent:	4.2	3	5	4.1	70%
4. Converting Percents to Decimals:	4.2	3	5	4.1	70%
Ratio and Proportion					
1. Definition of Ratio:	3.8	3.3	5	4	50%
2. Principle of Gears:	3.8	3.3	5	4	50%
3. The English System of Weights and Measures:	4.6	4.6	5	4.7	100%
4. The Metric of Weights and Measures:	3	4.6	5	4.2	70%

	μ V	μ M	μ A	μtot	%4-5
5. English-Metric Conversion:	4.2	4.6	5	4.6	80%
6. Multiplying and Dividing Measurements:	4.6	4.6	5	4.7	100%
7. Tables of Equivalent Measurements:	3	4.6	5	4.2	80%
Instruments of Measuring					
1. Linear Scales:	3.8	4.3	4.1	4.1	80%
2. The Gauge Number of Wire:	4	4.6	4.7	4.4	90%
Length					
1. Basic Operations Using Length:	4	5	5	4.7	90%
2. Length as a Measure of Distance:	4	5	5	4.7	90%
3. Calculations of a Rectangle (Perimeter, Width & Length):	3.6	4.3	4.3	4.1	90%
Area					
1. As a Measure of Surface Space:	5	4	4	4.3	90%
2. Calculations of Four-Sided Figures (Parallelogram, Trapezoid, Square, and Rectangle):	5	4.2	3.3	4.2	80%
3. Area Calculations for a Circle:	5	3.8	4	4.3	80%
Volume					
1. Measures of Volume:	5	4	4	4.3	80%
2. Volume Determined by Area:	5	4.2	3.3	4.2	60%
Weight					
1. Specific Gravity:	5	3.4	3.7	4.3	50%
2. Density:	5	4	4	4.3	80%

	μ V	μ M	μ A	μtot	%4-5
3. Weight Standards:	5	4.2	4.3	4.5	80%
4. Weights of Materials:	5	3.8	4	4.3	80%
Time					
1. Frequency:	5	4	3.7	4.2	80%
Money Calculations					
1. Price Computations:	5	5	3	4.3	80%
2. Income Computations:	5	4.2	3.3	4.2	80%
BASIC ALGEBRA					
Work and Power					
1. Electrical Power-Watt:	5	4.4	4	4.5	82%
2. Mechanical Efficiency of Machines:	5	4.2	3.3	4.2	73%
THE D. C. CIRCUIT					
1. Difference of Potential and Electric Current:	4.5	5	5	4.8	100%
2. E.M.F. and Energy Considerations:	4.5	4.7	4	4.4	89%
3. Ohm's Law:	4.5	5	5	4.8	100%
4. Voltage Drops:	4.5	5	5	4.8	100%
5. Energy and Power Dissipated in a Circuit:	4.5	5	5	4.8	100%
6. Series and Parallel Circuits; Equivalent Resistance:	4.5	4.7	5	4.7	100%
7. Direction of Currect:	4.5	4.5	4.3	4.4	78%
8. Kirchhoff's Rules:	4.5	4.2	4.3	4.3	78%
9. The Wheatstone Bridge:	3.5	4.5	4	4	78%

FUNDAMENTAL ELEMENTS IN AN A.C. CIRCUIT

	μ V	μ M	μ A	μtot	%4-5
1. Capacitance:	4	4.7	5	4.6	89%
2. Capacitors in Series:	4	4.7	5	4.6	89%
3. Capacitors in Parallel:	4	4.7	5	4.6	89%
4. Energy Required to Charge a Capacitor:	4	4	4.3	4.1	67%
5. Inductance:	4	4	4.7	4.2	78%
6. Phase Relation in A.C. Circuits:	4.5	4.5	5	4.7	89%
7. The Series A.C. Circuits; Phase Diagram:	4.5	3.8	4.7	4.3	78%
8. Reactance and Impedance:	4	4.2	4.7	4.3	78%
9. Power and Power Factor:	4.5	4.5	4.7	4.6	100%

Welding

	μ V	μ M	μ A	μtot	%4-5
BASIC ARITHMETIC					
Real Numbers					
1. Basic Operations with Whole Numbers:	5	5	4.3	4.8	88%
2. Combined Operations with Whole Numbers:	4.8	5	4.3	4.7	88%
3. Rounding Off:	4.8	5	3	4.3	75%
4. Prime Numbers:	4.3	5	3	4.1	63%
5. Greatest Common Diviser:	4.3	5	4	4.4	75%
6. Least Common Multiple:	4	5	4	4.3	63%
7. Absolute Value:	4.3	5	3	4.1	63%
Common Fractions					
1. Reading and Writing:	5	5	4.7	4.9	100%
2. Reading a Ruler:	5	5	4.7	4.9	100%
3. Basic Operations with Fractions:	5	5	4.3	4.8	88%
4. Equivalent Fractions:	5	5	3.3	4.4	86%
5. Converting Mixed Numbers to Improper Fractions:	5	5	3.3	4.4	86%
6. Basic Operations with Mixed Numbers:	5	5	4.3	4.8	86%
7. Combined Operations with Mixed Fractions:	5	5	3.7	4.6	86%
8. Proper and Improper Fractions:	5	5	3.7	4.6	86%
9. Complex Numbers:	5	5	3	4.3	86%
10. Combining Like Terms:	5	5	3	4.3	86%
11. Reducing to Lowest Terms:	5	5	3.7	4.6	86%
12. Finding the Lowest Common Denominator:	5	5	3.3	4.4	86%

	μ V	μ M	μ A	μtot	%4-5
13. Basic Operations with Common Fractions and Mixed Fractions:	5	5	3.7	4.6	86%
Decimal Fractions					
1. Reading and Writing:	5	5	4.3	4.8	100%
2. Rounding Off:	5	5	3	4.3	71%
3. Basic Operations:	5	5	3.3	4.4	71%
4. Table of Decimal Equivalents:	5	4	3.3	4.1	57%
5. Fraction to Decimal Conversion:	5	5	3.7	4.6	86%
6. Decimal to Fraction Conversion:	5	5	4	4.7	86%
Squares and Square Roots					
1. Using the Table of Square Roots:	4.8	5	3.3	4.4	86%
2. Finding the Square of a Number:	4.8	5	3.3	4.4	86%
3. Finding the Square Root of a Number:	4.8	5	3.3	4.4	86%
4. The Pythagorean Theorem:	4	5	3.3	4.1	71%
Percentage					
1. Definition:	5	5	3.7	4.6	86%
2. Converting Fractions to Percent:	4	5	3.3	4.1	63%
3. Converting Percent to Fractions:	4	5	3.3	4.1	63%
4. Converting Decimals to Percent:	4	5	3.3	4.1	63%
5. Finding the Percentage Given the Base and Rate:	4	5	3.7	4.2	63%

	µ V	µ M	µ A	µtot	%4-5
6. Finding the Rate Given the Percentage and Base:	4	5	3.7	4.2	63%
7. Finding the Base Given the Rate and Percentage:	4	5	3.7	4.2	63%
8. Solving Problems:	4.3	5	4.3	4.5	75%
9. Price Lists and Discount:	4	5	3.3	4.1	63%
10. Percentage Error and Averaging:	4.3	5	3	4.1	63%
Ratio and Proportion					
1. Definition of Ratio:	4	5	3.7	4.2	71%
2. Reduction of Ratio to Lowest Terms:	3.8	5	3.3	4	63%
3. Definition of Proportion:	3.8	5	3.7	4.2	63%
4. Finding the Unknown Term in a Proportion:	3.8	5	3.3	4	63%
5. Application of Proportion:	4.3	5	3.7	4.3	75%
6. The Principle of Levers:	3.5	5	3.7	4.1	50%
7. The Principle of Pulleys:	3.5	5	3.7	4.1	50%
8. The English System of Weights and Measures:	4.3	5	4.3	4.5	63%
9. The Metric of Weights and Measures:	4.3	5	3.3	4.2	63%
10. Multiplying and Dividing Measurements:	4.8	5	3.7	4.5	75%
11. Tables of Equivalent Measurements:	4.3	5	3.3	4.2	75%
Instruments of Measuring					
1. Linear Scales:	5	5	3.3	4.4	75%
2. The Protractor:	5	5	3.3	4.4	75%
3. The Framing Square:	5	3	4.3	4.1	75%

	μ V	μ M	μ A	μtot	%4-5
Length					
1. Baisc Operations Using Length:	5	5	3.3	4.4	88%
2. Length as a Measure of Distance:	5	5	3.7	4.6	88%
3. Calculations of a Rectangle (Perimeter, Width, Length):	5	5	3	4.3	75%
4. Calculations of a Triangle and Auxiliary Lines:	5	5	3	4.3	75%
5. Calculations of a Circle:	5	5	3.7	4.6	75%
6. Finding the Diameter of a Circle when the Area is Given:	5	5	3.7	4.6	75%
7. The Perimeter of the Ellipse:	5	5	2	4	75%
Area					
1. As a Measure of Surface Space:	5	5	3.7	4.6	75%
2. Calculations of Four-Sided Figures (Parallelogram, Trapezoid, Square and Rectangle):	5	5	2	4	75%
3. Area Calculations for a Circle:	5	5	3	4.3	75%
4. Relationship of Radius, Diameter, Circumference, Arc, Sectors, Chords, and Tangents:	5	5	3.7	4.6	88%
5. The Properties of π:	5	5	3.3	4.4	75%
6. Area of a Regular Hexagon:	5	5	2	4	75%
Volume					
1. Measures of Volume:	5	5	3	4.3	71%
2. Volume Determined by Area:	5	5	3	4.3	71%

	µ V	µ M	µ A	µtot	%4-5
3. Volume of Rectangular Solids (Cubes):	5	5	3	4.3	71%
Weight					
1. Measures of Mass:	5	5	2.3	4.1	57%
2. Weight Standards:	5	5	2.7	4.2	71%
3. Weights of Materials:	5	5	3.3	4.4	71%
Money Calculations					
1. Price Computation:	4.5	5	4.3	4.6	75%
2. Income Computation:	4.5	5	3	4.2	63%
3. Interest Computation:	4.5	5	2.7	4.1	63%
Scale Drawings					
1. Finding the Drawing Measure:	5	5	4	4.7	88%
2. Finding the Actual Dimensions of an Object from the Measured Dimension on a Diagram:	5	5	3.7	4.6	88%
3. Finding the Area of a Figure Drawn to a Certain Scale:	5	5	3	4.3	75%
BASIC ALGEBRA					
General Numbers					
1. Properties of Real Numbers:	5	4.7	4	4.6	85%
2. Translation of Word Expressions into Algebraic Expressions:	5	3	4	4	65%
3. Order of Operations:	5	3	4	4	65%
4. Evaluating Expressions Containing Variables:	5	3	4	4	68%

		μ V	μ M	μ A	μtot	%4-5
5.	Basic Operations Involving Variables:	5	3	4	4	68%
4.	Powers and Exponents:	4.5	3.7	4.5	4.2	79%
5.	Ratio and Proportion:	5	3.7	4.5	4.4	90%
	Arithmetic Operations with Algebraic Expressions					
1.	Basic Operations with Algebraic Numbers:	5	4.5	3	4.2	80%
2.	Operations with Signs of Grouping:	5	4.5	3	4.2	80%
3.	Distributive Law:	5	4.5	3	4.2	80%
4.	Equations Involving Paretheses:	5	4.5	3	4.2	80%
5.	Equations as Symbolic Sentences:	5	4.5	3	4.2	78%
6.	Removing Parentheses Used to Indicate Multiplication:	5	4.1	3	4	76%
7.	First Degree Equations with Signed Numbers:	5	4.1	3	4	76%
8.	Adding and Subtracting Like Terms:	5	4.5	3.5	4.3	79%
9.	Adding and Subtracting Polynomials:	5	4.5	3	4.2	88%
10.	Multiplication and Division of Monomials and Polynomials:	5	4.5	3	4.2	79%
11.	Inserting Signs of Grouping:	5	4.5	3	4.2	82%
12.	Rules of Exponents:	5	4.5	3	4.2	76%
13.	The Square of a Binomial Sum or Difference:	5	4.2	3	4.1	74%
14.	The Product of a Binomial Sum or Difference:	5	4.1	3	4	65%
15.	Multiplication by a Common Factor:	5	4.2	3	4.1	75%

		μ V	μ M	μ A	μtot	%4-5
16.	Special Products or Expansions:	5	4.2	3.5	4.2	75%
17.	Extracting Square Roots:	5	4	4.5	4.5	78%
18.	Symbols of Grouping:	5	4	3	4	62%

Machine Shop

	µ V	µ M	µ A	µtot	%4-5
BASIC ARITHMETIC					
Real Numbers					
1. Basic Operations with Whole Numbers:	5	5	5	5	100%
2. Combined Operations with Whole Numbers:	5	5	5	5	100%
3. Definition of Signed Numbers:	5	5	5	5	100%
4. Graphing Signed Numbers:	5	5	5	5	100%
5. Basic Operations with Signed Numbers:	4.7	5	5	4.9	94%
6. Sets:	4	5	5	4.7	90%
7. Rounding Off:	4.7	5	5	4.9	88%
8. Prime Numbers:	4	5	4.5	4.5	88%
9. Composite Numbers:	4	4	4.5	4.2	77%
10. Greatest Common Divisor:	4.7	4	4	4.2	66%
11. Least Common Multiple:	4.7	4	4	4.2	66%
12. Absolute Value:	4	4	4.1	4	75%
Common Fractions					
1. Reading and Writing:	5	5	5	5	100%
2. Reading a Ruler:	5	5	5	5	100%
3. Basic Operations with Fractions:	5	5	5	5	100%
4. Equivalent Fractions:	5	5	5	5	100%
5. Converting Mixed Numbers to Improper Fractions:	5	5	5	5	100%
6. Basic Operations with Mixed Numbers:	5	5	5	5	100%
7. Combined Operations with Mixed Numbers:	5	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
8. Proper and Improper Fractions:	5	5	5	5	100%
9. Complex Numbers:	4.7	3	5	4.2	89%
10. Combining Like Terms:	4.5	5	5	4.8	90%
11. Reducing to Lowest Terms:	4.7	5	5	4.9	100%
12. Finding the Lowest Common Denominator:	4.7	5	5	4.9	100%
13. Basic Operations with Common and Mixed Fractions:	5	5	5	5	100%
Decimal Fractions					
1. Reading and Writing:	5	5	5	5	100%
2. Rounding Off:	4	5	5	4.7	89%
3. Basic Operations:	5	5	5	5	100%
4. Table of Decimal Equivalents:	4.6	5	5	4.9	89%
5. Fraction to Decimal Conversion:	5	5	5	5	100%
6. Decimal to Fraction Conversion:	5	5	5	5	100%
Scientific Notation					
1. The Concept of Powers of 10:	3.6	5	5	4.5	75%
2. Significant Figures:	4	5	5	4.7	88%
3. Rounding Off:	3.8	5	5	4.6	87%
4. Basic Operations Using Powers of 10:	3.6	5	5	4.5	62%
5. Raising to Powers Using Powers of 10:	3.6	5	5	4.5	71%
6. Extracting Square Roots Using Powers of 10:	3.6	5	5	4.5	71%

	μ V	μ M	μ A	μtot	%4-5
Squares and Square Roots					
1. Finding the Square of a Number:	4.2	5	5	4.7	88%
2. Finding the Square Root of a Number:	4.2	5	5	4.7	88%
3. Finding the Square Root by the Long Division Method:	3.6	3.5	5	4	64%
4. Basic Operations with Radicals:	3.6	3.5	5	4	64%
5. The Pythagorean Theorem:	3.6	5	5	4.5	78%
6. Rationalization:	3.6	5	4.5	4.4	79%
Percentage					
1. Definition:	4.6	5	5	4.9	100%
2. Converting Fractions to Percent:	4.6	5	5	4.9	100%
3. Converting Percents to Fractions:	4.6	5	5	4.9	100%
4. Converting Decimals to Percents:	4.6	5	5	4.9	100%
5. Converting Percents to Decimals:	4.6	5	5	4.9	100%
6. Solving Problems:	4.6	5	5	4.9	100%
7. Price Lists and Discount:	3.4	5	4	4.1	50%
8. Percentage Error and Averaging:	3.4	4	3.5	4	50%
9. Markup:	3.2	5	4	4.1	50%
Ratio and Proportion					
1. Definition of Ratio:	5	5	5	5	100%
2. Reduction of Ratio to Lowest Terms:	4.9	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
3. Definition of Proportion:	4.9	5	5	5	100%
4. Finding the Unknown Term in a Proportion:	4.6	5	5	4.9	92%
5. Application of Proportion:	4.9	5	5	5	100%
6. The Principle of Levers:	4.6	5	3	4.2	88%
7. The Principle of Pulleys:	4.6	5	4	4.5	88%
8. The Principle of Gears:	4.6	5	4	4.5	88%
9. Principle of the Hydraulic Press:	4.6	5	4	4.5	88%
10. The English System of Weights & Measures:	4.3	5	4.5	4.6	100%
11. The Metric System of Weights & Measures:	4.9	5	4.5	4.8	100%
12. English-Metric Conversion:	4.9	5	4	4.6	88%
13. Multiplication & Division Measurements:	4.6	5	5	4.9	100%
14. Tables of Equivalent Measurements:	4.6	5	4	4.5	100%
Instruments of Measuring					
1. Linear Scales:	5	5	4	4.7	100%
2. Vernier Calipers:	4.6	5	4	4.5	94%
3. Micrometers:	4.9	5	4	4.6	100%
4. The Protractor:	5	3	4	4	94%
Length					
1. Calculations of a Triangle and Auxiliary Lines:	5	3	4	4	82%
2. Calculations of a Circle:	4.6	5	5	4.9	94%
3. Finding the Diameter of a Circle When the Area is Given:	4.6	5	5	4.9	94%

	μ V	μ M	μ A	μtot	%4-5
4. The Perimeter of the Ellipse:	3.4	5	4.5	4.3	67%
5. The Perimeter of Polygons:	3.8	5	4	4.3	67%
Area					
1. As a Measure of Surface Space:	3.8	5	3.5	4.1	67%
2. Calculations of Four-Sided Figures (Parallelogram, Trapezoid, Square, and Rectangle):	3.6	5	3.5	5	56%
3. Area Calculations for a Circle:	4.6	5	5	4.9	89%
4. Relationship of Radius, Diameter, Circumference, Arcs, Sectors, Chords, and Tangents:	4.6	5	5	4.9	89%
5. The Properties of π:	5	5	5	5	100%
6. Surface Area of Solids:	5	3	4	4	67%
7. Area Approximation:	5	4	3	4	67%
Volume					
1. Measures of Volume:	5	4	3	4	67%
2. Volume Determined by Area:	5	4.2	3	4.1	78%
3. Volume of Rectangular Solids (Cubes):	5	4	3	4	67%
4. Volume of a Sphere:	5	4	3	4	67%
5. Volume of a Cylinder:	5	3.8	3.3	4	70%
6. Heights of Cylinders:	5	4	3	4	78%
Weight					
1. Measures of Mass:	3.6	5	4	4.2	67%
2. Specific Gravity:	4.1	5	4	4.4	72%

	μ V	μ M	μ A	μtot	%4-5
3. Density:	3.8	5	4	4.3	71%
4. Weight Standards:	3.8	5	4	4.3	71%
5. Weights of Materials:	4.2	5	3	4.1	78%
6. Weight of Castings from Patterns:	4.2	5	3	4.1	78%
Time					
1. Units of Time:	4	5	5	4.7	92%
2. Distance in Relation to Time:	3.6	5	5	4.5	94%
3. Period:	3.8	5	5	4.6	84%
4. Frequency:	3.8	5	5	4.6	84%
Money Calculations					
1. Income Computation:	5	3.6	3.5	4	67%
2. Interest Computation:	5	3.6	3.5	4	67%
Scale Drawings					
1. Finding the Drawing Measure:	4.4	5	5	4.8	84%
2. Finding the Actual Dimensions of an Object from the Measured Dimension of a Diagram:	4.4	5	5	4.8	84%
3. Finding the Area of a Figure Drawn to a Certain Scale:	4.4	5	5	4.8	84%
Geometry					
1. Straight Line & Circle:	4.6	3	5	4.2	92%

BASIC ALGEBRA

	μ V	μ M	μ A	μtot	%4-5
General Numbers					
1. Properties of Real Numbers:	3.8	5	3.5	4.1	78%
2. Substitution of Variables into Algebraic Expressions:	4.2	5	5	4.7	72%
3. Translation of Word Expressions into Algebraic Expressions:	4	5	5	4.7	73%
4. Order of Operations:	4	5	5	4.7	74%
5. Evaluating Expressions Containing Variables:	4.2	5	5	4.7	78%
6. Basic Operations Involving Variables:	4.2	5	5	4.7	79%
7. Axioms:	3.8	5	5	4.6	92%
8. Constants and Variables:	4.4	5	5	4.8	92%
9. Positive and Negative Numbers:	4.8	5	5	4.9	100%
10. Powers and Exponents:	4.2	5	5	4.7	100%
11. Formulas:	4.6	5	5	4.9	100%
12. Ratio and Proportion:	4.8	5	5	4.9	100%
Arithmetic Operations with Algebraic Expressions					
1. Basic Operations with Algebraic Numbers:	4.2	5	5	4.7	94%
2. Operations with Signs of Grouping:	3.8	5	5	4.6	95%
3. The Distributive Law:	3.8	5	5	4.6	93%
4. Equations Involving Parentheses:	4.4	5	5	4.8	94%
5. Equations as Symbolic Sentences:	3.8	5	5	4.6	86%

	μ V	μ M	μ A	μtot	%4-5
6. Equations Involving Parentheses:	4.4	5	5	4.8	94%
7. Equations as Symbolic Sentences:	3.8	5	5	4.6	86%
8. Removing Parentheses Used to Indicate Multiplication:	4	5	5	4.7	82%
9. First Degree Equations with Signed Numbers:	3.8	5	5	4.6	94%
10. Adding and Subtracting Like Terms:	4.6	5	5	4.9	100%
11. Adding and Subtracting Polynomials:	4.4	5	5	4.8	100%
12. Multiplication and Division of Monomials and Polynomials:	4.2	3	5	4.1	78%
13. Inserting Signs of Grouping:	4	5	5	4.7	94%
14. Rules of Exponents:	4	5	5	4.7	94%
15. The Square of a Binomial Sum or Difference:	3.8	5	5	4.6	88%
16. The Product of a Binomial Sum or Difference:	3.6	5	5	4.5	94%
17. Multiplication by a Common Factor:	4	5	5	4.7	94%
18. Extracting Square Roots:	4	5	5	4.7	94%
19. Symbols of Grouping:	3.8	5	5	4.6	90%

Algebraic Fractions

1. Properties of Algebraic Fractions:	4.2	5	5	4.7	96%
2. Equations Involving Fractions:	4.8	5	5	4.9	100%
3. Basic Operations with Algebraic Fractions:	4.5	5	5	4.8	100%

	μ V	μ M	μ A	μtot	%4-5
5. Finding the Least Common Denominator:	4.4	3	5	4.1	88%
6. Reducing to Lower Terms:	4.6	3	5	4.2	94%
Inequalities					
1. Properties of Inequalities:	3.1	5	5	4.4	82%
Equations					
1. First Degree Equations in One and Two Variables:	3.8	5	5	4.6	80%
2. Literal Equation Formulas:	3.5	5	5	4.5	82%
3. Equivalent, Identical, and Conditional Equations:	4	5	3	4	63%
4. Simplification of Equations:	3.3	5	5	4.4	80%
5. Solutions of Equations in Variables:	3.3	5	5	4.4	82%
6. Transposition of Terms and Factors:	3.3	5	5	4.4	82%
7. Translating Problems into Equations:	3.5	5	5	4.5	84%
8. Operations with Dimensional Units:	3.5	5	5	4.5	84%
9. Dimensional Units and Conversion:	4	5	5	4.7	87%
10. Equations Containing Fractions and Decimals:	4	4.5	5	4.5	82%
11. Solving Linear Equations:	3.6	5	5	4.5	82%
12. Proportions:	3.2	5	5	4.4	72%
13. Proportionality:	3.2	5	5	4.4	72%
Linear Graphs					
1. Coordinate Notation:	3	5	5	4.3	68%

	μ V	μ M	μ A	μtot	%4-5
2. Plotting Points on Graphs:	3.5	5	5	4.5	72%
3. Graphs of Ordered Pairs:	2.9	5	5	4.3	66%
4. Displaying Data with Graphs:	3.5	5	5	4.5	76%
5. Solving Problems by Means of Graphs:	3.2	5	5	4.4	70%
6. Graphs of Linear Equations:	3.2	5	5	4.4	71%

Printing

	μ V	μ M	μ A	μtot	%4-5
BASIC ARITHMETIC					
Real Numbers					
1. Basic Operations with Whole Numbers:	5	5	5	5	100%
2. Combined Operations with Whole Numbers:	5	5	5	5	100%
3. Definition of Signed Numbers:	5	5	4	4.7	100%
4. Graphing Signed Numbers:	5	5	3	4.3	83%
5. Basic Operations with Signed Numbers:	5	5	3	4.3	83%
6. One-to-One Correspondence:	4.3	3	5	4.1	66%
7. Rounding Off:	4.7	5	4	4.6	100%
8. Prime Numbers:	4.7	5	4	4.6	100%
9. Composite Numbers:	4.3	5	3	4.1	66%
10. Greatest Common Diviser:	4	5	5	4.7	83%
11. Least Common Multiple:	4	5	4	4.3	83%
Common Fractions					
1. Reading and Writing:	5	5	5	5	100%
2. Reading a Ruler:	5	3.5	5	4.5	83%
3. Basic Operations with Fractions:	5	5	5	5	100%
4. Equivalent Fractions:	4.7	5	4	4.6	100%
5. Converting Mixed Numbers to Improper Fractions:	4.7	5	3	4.2	83%
6. Basic Operations with Mixed Numbers:	4.7	5	4	4.6	100%
7. Combined Operations with Mixed Fractions:	4.7	5	4	4.6	100%
8. Proper and Improper Fractions:	4.7	5	4	4.6	100%

	μ V	μ M	μ A	μtot	%4-5
9. Combining Like Terms:	4.7	5	4	4.6	100%
10. Reducing to Lowest Terms:	5	5	3	4.3	83%
11. Finding the Lowest Common Denominator:	4.7	5	4	4.6	100%
12. Basic Operations with Common Fractions and Mixed Fractions:	5	5	3	4.3	83%
Decimal Fractions					
1. Reading and Writing:	5	5	4	4.7	100%
2. Rounding Off:	5	5	4	4.7	100%
3. Basic Operations:	5	5	4	4.7	100%
4. Table of Decimal Equivalents:	4.7	5	4	4.6	100%
5. Fraction to Decimal Conversion:	5	5	3	4.3	83%
6. Decimal to Fraction Conversion:	5	5	3	4.3	83%
Scientific Notation					
1. The Concept of Powers of 10:	5	5	2	4	83%
2. Significant Figures:	5	5	2	4	83%
3. Rounding Off:	5	5	2	4	83%
4. Basic Operations Using Powers of 10:	5	5	2	4	83%
Percentage					
1. Definition:	5	5	4	4.7	100%
2. Converting Fractions to Percents:	5	5	4	4.7	100%
3. Converting Percents to Fractions:	5	5	3	4.3	83%

	μ V	μ M	μ A	μtot	%4-5
4. Converting Decimals to Percents:	5	5	3	4.3	83%
5. Converting Percents to Decimals:	5	5	3	4.3	83%
6. Solving Problems:	5.7	5	3	4.6	83%
7. Price Lists and Discount:	4.3	5	5	4.8	83%
8. Percentage Error and Averaging:	4.3	5	4	4.4	83%
9. Markup:	4.3	5	5	4.8	83%
Ratio and Proportion					
1. Definition of Ratio:	5	4.5	3	4.2	83%
2. Reduction of Ratio to Lowest Terms:	5	4.5	3	4.2	83%
3. Definition of Proportion:	5	4.5	3	4.2	83%
4. Finding the Unknown Term in a Proportion:	5	4.5	3	4.2	83%
5. Application of Proportion:	5	4.5	3	4.2	83%
6. The English System of Weights and Measures:	5	5	5	5	100%
7. The Metric of Weights and Measures:	4.7	5	5	4.9	100%
8. English-Metric Conversion:	4.7	5	5	4.9	100%
9. Multiplying and Dividing Measurements:	4.7	5	5	4.9	100%
10. Tables of Equivalent Measurements:	4.7	5	5	4.9	100%
Instruments of Measuring					
1. Linear Scales:	5	5	4	4.7	100%
2. Vernier Calipers:	4.3	4.5	5	4.6	100%
3. The Protractor:	4.3	4.5	5	4.3	100%

	μ V	μ M	μ A	μtot	%4-5
Length					
1. Basic Operations Using Length:	5	5	5	5	100%
2. Calculations of a Triangle and Auxiliary Lines:	5	5	2	4	83%
3. Calculations of a Circle:	5	5	2	4	83%
Area					
1. As a Measure of Surface Space:	5	5	2	4	83%
2. Calculations of Four-Sided Figures (Squares, Rectangles, etc...)	5	5	2	4	83%
Volume					
1. Measures of Volume:	5	5	3	4.3	83%
Weight					
1. Measures of Mass:	4.7	5	3	4.2	83%
2. Weight Standards:	5	5	4	4.7	100%
3. Weights of Materials:	5	5	5	5	100%
Time					
1. Units of Time:	5	5	5	5	100%
2. Frequency:	5	5	2	4	75%
3. Period:	5	5	2	4	75%
Money Calculations					
1. Price Computations:	5	5	4	4.7	100%
2. Income Computations:	5	5	4	4.7	100%
3. Interest Computations:	5	5	4	4.7	100%

	μ V	μ M	μ A	μtot	%4-5
Slide Rule					
1. Slide Rule Operations (Basic):	4.3	5	3	4.1	67%
2. Locating the Decimal Point:	4.7	5	5	4.9	100%

Drafting

	μ V	μ M	μ A	μtot	%4-5
BASIC ARITHMETIC					
Real Numbers					
1. Basic Operations with Whole Numbers:	5	5	5	5	100%
2. Combined Operations with Whole Numbers:	5	5	5	5	100%
3. Definition of Signed Numbers:	5	5	5	5	100%
4. Graphing Signed Numbers:	4.3	5	5	4.8	100%
5. Basic Operations with Signed Numbers:	5	5	5	5	100%
6. Rounding Off:	4.5	5	5	4.8	100%
7. Prime Numbers:	4	5	3	4	50%
8. Composite Numbers:	4.5	5	3	4.2	83%
9. Greatest Common Divisor:	4	5	4	4.3	67%
10. Least Common Multiple:	4	5	4	4.3	67%
11. Absolute Value:	4.3	4	4	4.1	100%
Common Fractions					
1. Reading and Writing:	4.5	5	5	4.8	83%
2. Reading a Ruler:	5	5	5	5	100%
3. Basic Operation with Fractions:	5	4	5	4.7	100%
4. Equivalent Fractions:	4.8	5	4	4.6	100%
5. Converting Mixed Numbers to Improper Fractions:	4.8	5	4	4.6	100%
6. Basic Operations with Mixed Numbers:	4.5	5	5	4.8	83%
7. Combined Operations with Mixed Fractions:	5	5	5	5	100%
8. Proper and Improper Fractions:	4.5	5	4	4.5	83%

	μ V	μ M	μ A	μtot	%4-5
9. Combining Like Terms:	4.8	5	5	4.9	100%
10. Reducing to Lowest Terms:	4.8	5	5	4.9	100%
11. Finding the Lowest Common Denominator:	4.8	5	5	4.9	100%
12. Basic Operations with Common Fractions and Mixed Fractions:	4.8	5	5	4.9	100%
Decimal Fractions					
1. Reading and Writing:	5	5	5	5	100%
2. Rounding Off:	5	5	5	5	100%
3. Basic Operations:	5	5	5	5	100%
4. Table of Decimal Equivalents:	4.5	5	5	4.8	83%
5. Fraction to Decimal Conversion:	4	5	5	4.7	67%
6. Decimal to Fraction Conversion:	3.5	5	5	4.5	67%
Scientific Notation					
1. The Concept of Powers of 10:	4.5	5	4	4.5	100%
2. Significant Figures:	4.8	5	4	4.6	100%
3. Rounding Off:	4.5	5	5	4.8	100%
4. Basic Operations Using Powers of 10:	4.8	5	4	4.6	100%
5. Raising to Powers Using Powers of 10:	4.8	5	4	4.6	100%
6. Extracting Square Roots Using Powers of 10:	4.5	5	5	4.8	100%
Squares and Square Roots					
1. Using the Table of Square Roots:	4.5	3	5	4.2	83%

	μ V	μ M	μ A	μtot	%4-5
2. Finding the Square of a Number:	5	4	4	4.3	100%
3. Finding the Square Root of a Number:	5	5	3	4.3	83%
4. The Pythagorean Theorem:	5	5	3	4.3	83%
Percentage					
1. Definition:	4.3	5	3	4.1	67%
2. Converting Fractions to Percent:	4.5	5	3	4.2	83%
3. Solving Problems:	4.5	5	3	4.2	67%
Ratio and Proportion					
1. Definition of Ratio:	5	5	3	4.3	83%
2. Definition of Proportion:	5	5	2	4	83%
3. Finding the Unknown Term in a Proportion:	5	5	2	4	83%
4. Application of Proportion:	4.8	5	3	4.3	83%
5. The English System of Weights and Measures:	4.5	5	3	4.2	67%
6. Multiplying and Dividing Measurements:	4.8	5	3	4.2	83%
7. Tables of Equivalent Measurements:	4.3	5	4	4.4	83%
Instruments of Measuring					
1. Linear Scales:	5	5	5	5	100%
2. Vernier Calipers:	4.3	5	5	4.8	83%
3. Micrometers:	4.3	4	3	4.1	67%
4. The Protractor:	4.8	5	5	4.9	100%
5. The Planimeters:	3.8	5	5	4.6	83%

	μ V	μ M	μ A	μtot	%4-5
6. Logarithmic Scales:	3.5	5	4	4.2	67%
7. Angular Scales:	4.8	5	5	4.9	100%
8. Uniform Scales:	3.5	5	4	4.2	67%
9. The Circumference Rule:	3.5	5	4	4.2	67%
Length					
1. Basic Operations Using Length:	5	5	3	4.3	83%
2. Length as a Measure of Distance:	5	5	4	4.7	100%
3. Calculations of a Rectangle (Perimeter, Width, Length):	5	5	5	5	100%
4. Calculations of a Triangle and Ausiliary Lines:	5	5	5	5	100%
5. Calculations of a Circle:	5	5	5	5	100%
6. Finding the Diameter of a Circle when the Area is Given:	5	5	5	5	100%
7. The Perimeter of the Ellipse:	4	5	3.3	4.1	67%
8. The Perimeter of Polygons:	4.3	5	4	4.4	83%
Area					
1. As a Measure of Surface Space:	5	5	5	5	100%
2. Calculations of Four-Sided Figures (Parallelogram, Trapezoid, Square, and Rectangle):	4.3	5	5	4.8	83%
3. Area Calculations for a Circle:	5	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
4. Relationship of Radius, Diameter, Circumference, Arcs, Sectors, Chords, and Tangents:	4.7	5	5	4.9	100%
5. The Properties of π:	4	5	5	4.7	83%
6. Regular Polygons:	4	5	3	4	50%
7. Composite Figures:	3.8	5	4	4.3	67%
8. Area Calculations of Similar Figures:	4	5	3	4	67%
9. Surface Area of Solids:	4.5	5	4	4.5	100%
10. Area of Ring Sections (Sector and Segment):	3	5	4	4	67%
11. Area Approximation:	3.5	5	5	4.5	67%
Volume					
1. Measures of Volume:	5	5	5	5	100%
2. Volume Determined by Area:	5	5	5	5	100%
3. Volume of Rectangular Solids (Cubes):	5	5	5	5	100%
4. Volume of a Sphere:	4.5	5	3	4.2	83%
5. Segments and Zones of Spheres:	3.3	5	5	4.4	67%
6. Volume of a Cylinder:	4.8	5	3	4.3	83%
7. Surface Volume of Solid and Hollow Cylinders:	4.8	5	4	4.6	100%
8. Heights of Cylinders:	5	5	3	4.3	83%
9. Lateral Surface of Cylinders:	4.3	5	3	4.1	67%
10. Volume of Compound Figures:	4	5	3	4	50%
11. Volume of Composite Figures:	4	5	3	4	50%

	μ V	μ M	μ A	μtot	%4-5
Weight					
1. Weights of Materials:	4.8	4	4	4.3	100%
Time					
1. Units of Time:	4.8	5	4	4.6	100%
2. Distance in Relation to Time:	4.8	4	4	4.3	100%
Money Calculations					
1. Price Computation:	3.3	5	4	4.1	67%
2. Income Computation:	3.3	5	4	4.1	67%
3. Interest Computation:	3.3	5	4	4.1	67%
Scale Drawings					
1. Finding the Drawing Measure:	5	5	5	5	100%
2. Finding the Actual Dimensions of an Object From the Measured Dimension on a Diagram:	5	5	5	5	100%
3. Finding the Area of a Figure Drawn to a Certain Scale:	4	5	4	4.3	83%
Slide Rule					
1. Reading the C and D Scales:	2	5	5	4	33%
2. Combination Problems:	2	5	5	4	33%
3. Finding the Circumference and Area of a Circle:	2	5	5	4	33%

	μ V	μ M	μ A	μtot	%4-5
Geometry					
1. Straight Line & Circle:	4.8	5	4	4.6	100%
Geometrical Construction					
1. Bisect or Find Mid-Point of a Line:	4.8	5	5	4.9	100%
2. Bisect an Angle:	4.8	5	5	4.9	100%
3. Bisect an Arc:	4.8	5	5	4.9	100%
4. Erect a Perpendicular to a Line at a Given Point on the Line:	4.8	5	4	4.6	100%
5. Erect a Perpendicular to a Line at the End of the Line:	4.5	5	4	4.5	83%
6. Erect a Perpendicular Through a Point not on the Line:	4.3	5	4	4.4	83%
7. Draw a Line Through Given Points Parallel to a Given Line:	4.3	5	4	4.4	83%
8. Divide a Given Line into a Number of Equal Parts:	4.5	5	5	4.8	83%
9. Draw an Angle Equal to a Given Angle:	4.5	5	5	4.8	83%
10. Draw an Equilateral Triangle of Given Size:	4.5	5	5	4.8	83%
11. Draw a Circle Through Three Given Points:	4.5	5	3	4.2	67%
12. Find the Center of a Circle or Circular Arc:	4.5	5	5	4.8	83%
13. Inscribe a Square in a Circle:	4.5	5	4	4.5	83%
14. Construct a Square of Given Size:	4.5	5	5	4.8	83%
15. Inscribe a Hexagon in a Circle:	4.5	3	5	4.2	67%

	μ V	μ M	μ A	μtot	%4-5
16. Inscribe an Equilateral Triangle in a Circle:	4.5	3	5	4.2	67%
17. Construct a Hexagon with One of the Sides on a Given Line:	4.3	3	5	4.1	67%
18. Draw a Tangent to a Circle at a Point on the Circumference:	4.5	5	5	4.8	83%
19. Draw a Tangent to a Circle Through a Point Outside the Circle:	4.8	5	5	4.9	83%
20. Draw a Tangent to Two Circles of Equal Size:	4.3	5	5	4.9	83%
21. Draw an Internal Tangent to Two Circles of Equal Size:	3.8	5	5	4.6	67%
22. Draw an External Tangent to Two Circles of Unequal Size:	4.3	5	5	4.9	83%
23. Draw an Internal Tangent to Two Unequal Circles:	3.8	5	5	4.6	67%

BASIC ALGEBRA

General Numbers

	μ V	μ M	μ A	μtot	%4-5
1. Properties of Real Numbers:	4.3	5	5	4.9	80%
2. Substitution of Variables into Algebraic Expressions:	4.5	4	5	4.5	100%
3. Translation of Word Expressions into Algebraic Expressions:	5	4	5	4.7	100%
4. Order of Operations:	4.8	4	5	4.6	100%
5. Evaluating Expressions Containing Variables:	4.5	5	5	4.8	100%
6. Basic Operations Involving Variables:	4.5	5	4	4.5	100%

		μ V	μ M	μ A	μtot	%4-5
7.	Constants and Variables:	4.3	5	4	4.4	83%
8.	Positive and Negative Numbers:	5	5	4	4.7	100%
9.	Powers and Exponents:	5	5	2	4	83%
10.	Formulas:	4.5	5	5	4.8	83%
11.	Ratio and Proportion:	4.8	5	5	4.9	100%

Arithmetic Operations with Algebraic Expressions

		μ V	μ M	μ A	μtot	%4-5
1.	Basic Operations with Algebraic Numbers:	5	5	4	4.7	100%
2.	Operations with Signs of Grouping:	5	5	4	4.7	100%
3.	Equations Involving Parentheses:	4.8	5	3	4.3	83%
4.	Equations as Symbolic Sentences:	5	5	3	4.3	83%
5.	Removing Parentheses Used to Indicate Multiplication:	5	5	3	4.3	83%
6.	First Degree Equations with Signed Numbers:	5	5	3	4.3	83%
7.	Adding and Subtracting Like Terms:	5	5	4	4.7	100%
8.	Adding and Subtracting Polynomials:	5	5	4	4.7	100%
9.	Inserting Signs of Grouping:	4.8	5	4	4.6	100%
10.	Rules of Exponents:	4.8	5	4	4.6	100%
11.	Factoring Out the Common Factors:	5	4	3	4	83%
12.	Multiplication by a Common Factor:	5	3	4	4	83%
13.	Symbols of Grouping:	4.8	5	4	4.6	100%

	µ V	µ M	µ A	µtot	%4-5
Algebraic Fractions					
1. Properties of Algebraic Fractions:	4.5	4	4	4.2	100%
2. Equations Involving Fractions:	4.5	4	4	4.2	100%
3. Basic Operations With Algebraic Fractions:	4.3	4	4	4.1	100%
4. Finding the Least Common Denominator:	4.3	4	4	4.1	100%
5. Reducing to Lower Terms:	4	4	4	4	80%
Equations					
1. First Degree Equations in One and Two Variables:	4.8	5	3	4.3	80%
2. Literal Equation Formulas:	4.7	5	4	4.6	100%
3. Equivalent Identical and Conditional Equations:	4.3	5	4	4.4	100%
4. Simplification of Equations:	4.7	5	4	4.6	100%
5. Solution of Equations in Variables:	5	5	4	4.7	100%
6. Analysis of a Formula:	4.3	5	5	4.8	80%
7. Transposition of Terms:	5	5	5	5	100%
8. Transposition of Factors:	5	5	4	4.7	100%
9. Translating Problems into Equations:	5	5	5	5	100%
10. Operations with Dimensional Units:	5	5	5	5	100%
11. Solving Systems of Equations of Two and Three Unknowns Algebraically:	4	5	3	4	67%
12. Dimensional Units and Conversion:	4.3	5	4	4.4	100%

	μ V	μ M	μ A	μtot	%4-5
13. Equations Containing Fractions and Decimals:	4.3	5	4	4.4	100%
14. Degree of an Equation:	3.3	5	4	4.1	60%
15. Solving Linear Equations:	4.7	5	3	4.2	80%
16. Algebraic Solutions to Variables:	4.7	5	4	4.6	100%
17. Equations with Unknowns in the Denominators:	4.7	5	4	4.6	100%
18. Proportions:	4.7	5	5	4.9	100%
19. Proportionality:	4.3	5	5	4.8	80%

Linear Graphs

1. Coordinate Notation:	5	5	4	4.7	100%
2. Plotting Points on Graphs:	5	5	5	5	100%
3. Graphs of Ordered Pairs:	4.5	5	3	4.2	67%
4. Displaying Data with Graphs:	4.8	5	5	4.9	100%
5. Solving Problems by Means of Graphs:	4.5	5	4	4.5	100%
6. Graphs of Linear Equations:	4.5	5	4	4.5	100%
7. Determining Linear Function from Two Points:	4	5	3	4	67%
8. Graphing Parallel Lines:	4	5	3	4	60%
9. Graphing Perpendicular Lines:	4	5	3	4	60%
10. Graphing Lines Parallel to the Axis:	4	5	3	4	60%
11. Graphing Solutions of Simultaneous Equations:	4	5	4	4.3	80%
12. The Slope of a Line and the Y Intersect:	4.3	5	3	4.1	80%
13. The Distance Formula:	4	5	3	4	60%

	μ V	μ M	μ A	μtot	%4-5
14. Graphic Solution of System of Linear Equations:	4.3	4	4	4.1	80%
15. Solution by Addition and Subtraction:	4.7	5	4	4.6	100%
16. Applications of Simultaneous Linear Equations:	4.3	5	4	4.4	100%
Quadratic Equations					
1. Solution of $ax^2+bx+c=0$:	4.5	5	4	4.5	100%
2. Solution by the Quadratic Formula:	4.5	5	3	4.2	75%
Exponential Notation					
1. Laws of Basic Operations with Exponents:	4.5	5	4	4.5	83%
2. The Law of Zero Exponents:	4.3	5	3	4.1	67%
3. The Law of Exponents in Raising to Powers:	4.3	5	4	4.4	83%
4. The Law of Exponents in Extracting Roots:	4.3	5	3	4.1	67%
5. The Law of Exponents Based on 10:	4.3	5	5	4.8	83%
Radicals					
1. The Square Root of Positive Numbers:	4	5	3	4	83%
Logarithmic Notation					
1. Definition of Logarithms:	4.5	5	3	4.2	83%
2. Characteristics:	4.5	5	3	4.2	83%
3. Mantissa:	4.5	5	3	4.2	83%
4. Properties of Logarithms:	4.5	5	3	4.2	83%

	μ V	μ M	μ A	μtot	%4-5
5. Determining Logs from a Table:	4.5	5	4	4.5	100%
6. The Logarithm of Unity:	5	5	4	4.5	75%
7. Given n, Find Log n:	5	5	3.7	4.4	75%
8. Given Log n, Find n:	5	5	3.7	4.4	75%
ADVANCED ALGEBRA AND ANALYTICAL GEOMETRY					
Quadratic Equations and Formulas					
1. The Meaning of Quadratic Equations:	3.3	5	4	4.1	60%
Exponents					
1 The Meaning of Powers with Negative Whole-Number Exponents:	4	5	3	4	50%
2. The Reciprocals of Powers and Connecting Divisions to Multiplication:	4	5	3	4	50%
3. A Review of Powers and the Laws of Exponents:	4	5	3	4	67%
4. The Meaning of Roots and Radicals:	3.7	5	4	4.2	80%
Linear Graphs and Slope					
1. Graphing Linear Equations by Plotting Intercepts:	3.3	5	4	4.1	60%
2. A Graphical Representation of Changes in "y" Corresponding to Increases in "x":	3.7	5	4	4.2	80%
3. The Slope of a Line Expressed as a Signed Number:	4	5	4	4.3	100%
4. The Slope-Intercept Form of Linear Equations:	3.7	5	4	4.2	80%

	μ V	μ M	μ A	μtot	%4-5
5. Writing Linear Equations in Slope-Intercept Form:	3.7	5	4	4.2	80%
6. Slope and Parallel Lines:	3.7	5	4	4.2	80%
7. The Equations of Horizontal and Vertical Lines:	3.3	5	4	4.1	60%

TRIGONOMETRY

Right Triangles and Trigonometric Ratios

	μ V	μ M	μ A	μtot	%4-5
1. Right Triangles and Angle-Sum Principle:	4	4	4	4	83%
2. Right Triangles and the Pythagorean Theorem:	5	5	4	4.7	100%
3. The Definition of the Tangent Ratio:	4.3	5	4	4.4	80%
4. The Tangent Ratio of Three Specific Angles:	4	5	5	4.7	80%
5. A Partial Table of Tangent Ratios:	3.8	5	4	4.3	67%
6. The Complete Table of Trig Ratios:	4.5	5	5	4.8	83%
7. Finding the Unknown Sides in Right Triangles:	5	5	5	5	100%
8. Finding the Unknown Angles in Right Triangles:	5	5	5	5	100%

Vectors & Phasors

	μ V	μ M	μ A	μtot	%4-5
1. Definition of Vectors and Phasors:	4.3	5	4	4.4	83%
2. Vectors on the Rectangular Coordinate System:	4.5	5	4	4.5	83%
3. The Trig Ratios of Reference Angles:	3.3	5	4	4.1	60%
4. Finding the Length and Director of a Slanted Vector:	4	5	4	4.3	80%

	μ V	μ M	μ A	μtot	%4-5
Trigonometric Ratios of Standard-Position Angles					
1. Standard-Position Angles Between 0° and 360°:	3.3	5	4	4.1	60%
2. Reference Angles of Standard-Position Angles Between 0° and 360°:	3.3	5	4	4.1	60%
3. Trigonometric Ratios of Angles Between 90° and 360°:	3.3	5	4	4.1	60%
4. Finding the Function of any Angle:	4	5	5	4.7	80%
5. Finding Standard-Position Angles Corresponding to Known Ratios:	3.3	5	5	4.4	60%
6. Finding the Angle Given the Function:	4	5	5	4.7	80%
7. Trigonometric Ratios of 0°, 90°, 180°, 270°, and 360°:	3.3	5	4	4.1	60%
8. Functions of Angles in Quadrants I, II, III, and IV:	3.3	5	4	4.1	60%
9. Finding Log of Trig Functions:	3.3	5	4	4.1	60%
10. Definition of a "Degree" in Angle Measurements:	3	5	4	4	60%
11. Operations Using Minutes, Degrees & Seconds:	4.7	5	4	4.6	100%
12. Definition of Radian in Angle Measurement:	4.7	5	4	4.6	100%
13. The Relationship Between Degrees and Radians:	4.7	5	4	4.6	100%
14. Radians Expressed in Terms of "π":	4.7	5	3	4.2	80%
15. Trig Ratios of Angles Expressed in Radians:	4.7	5	3	4.2	80%

	μ V	μ M	μ A	μtot	%4-5
Oblique Triangles					
1. The Definition of an Oblique Triangle:	4.3	5	5	4.8	100%
2. Solving Oblique Triangles By Right-Triangle Methods:	4	4	4	4	60%
3. The Law of Sines:	4.3	5	4	4.4	100%
4. Using the Law of Sines to Find Unknown Angles Oblique Triangles:	3.7	5	4	4.2	80%
5. Limitations of Using the Law of Sines:	3.7	5	5	4.6	80%
6. The Law of Cosines:	3.7	5	5	4.6	80%
7. Using the Law of Cosines to Find Unknown Sides and Angles of Oblique Triangles:	3.7	5	4	4.2	80%
8. Right Triangles-Law of Sines and Law of Cosines:	3.3	5	5	4.4	60%
9. Strategies for Solving Triangles:	3.7	5	5	4.6	80%
10. The Law of Sines and Obtuse Angles:	3.7	5	5	4.6	80%
11. The Law of Cosines and Obtuse Angles:	3.7	5	5	4.6	80%
Applied Geometric Problems					
1. Central Angles and Arcs of Circles:	4.3	5	4	4.4	100%
2. Chords of Circles:	4	5	5	4.7	83%
3. Basic Problems Involving Chords and Circles:	4.5	5	5	4.8	83%
4. Applied Problems Involving Chords of Circles:	4.7	5	5	4.9	100%
5. Sectors and Segments of Circles:	4	5	3	4	50%

	μ V	μ M	μ A	μtot	%4-5
6. Revolutions and Rotational Speed:	4.3	5	3	4.1	67%
7. Circumferential Speed:	4.7	5	3	4.2	80%
8. Angular Measures of Rotational Speed:	4	5	3	4	67%
9. Circumferential Speed and Angular Velocity (In Radians):	4	5	3	4	60%
10. Areas of Triangles:	4.8	5	5	4.9	100%
Identities and Interpolation					
1. Definitions of the Cosecant, Secant, and Cotangents of an Angle:	3	5	5	4.3	40%
2. Finding the Cosecant, Secant, and Cotangent of an Angle:	3	5	5	4.3	40%
3. Ratio Identities	3	5	4	4	40%
4. Pythagorean Identities:	3.7	5	4	4.2	60%
5. Simple Identities:	3.3	5	4	4.1	60%
6. Reciprocal Identities:	3.3	5	4	4.1	60%
7. Verifying Identities:	3.3	5	5	4.4	60%
8. Similar Right Triangles:	3	5	4	4	60%
Numerical Trigonometry (Applications)					
1. Measurement of Angles:	5	5	5	5	100%
2. Mapping:	3.3	5	5	4.4	67%
3. Latitudes and Departures:	3.3	5	5	4.4	67%
4. Calculations of Areas:	5	5	5	5	100%
5. Measurement of Distances in a Vertical Plane:	5	5	5	5	100%

Electronics

	μ V	μ M	μ A	μtot	%4-5
BASIC ARITHMETIC					
Real Numbers					
1. Basic Operations with Whole Numbers:	5	5	5	5	100%
2. Combined Operations with Whole Numbers:	5	5	5	5	100%
3. Definition of Signed Numbers:	5	5	5	5	100%
4. Graphing Signed Numbers:	5	5	5	5	100%
5. Basic Operations with Signed Numbers:	5	5	5	5	100%
6. Rounding Off:	4.8	4.7	5	4.8	100%
7. Absolute Value:	4.5	5	5	4.8	100%
8. Greatest Common Diviser:	4.5	5	5	4.8	100%
9. Least Common Multiple:	4.5	5	5	4.8	100%
Common Fractions					
1. Reading and Writing:	3.5	4	5	4.2	67%
2. Reading a Ruler:	5	5	5	5	100%
3. Basic Operations with Fractions:	4.5	4.7	5	4.7	100%
4. Equivalent Fractions:	5	5	5	5	100%
5. Basic Operations with Mixed Numbers:	3.8	3.7	5	4.2	67%
6. Combined Operations with Mixed Numbers:	3.8	3.7	5	4.2	67%
7. Proper and Improper Fractions:	3.5	3.7	5	4.1	67%
8. Complex Numbers:	3	4	5	4	67%

	μ V	μ M	μ A	μtot	%4-5
9. Combining Like Terms:	5	5	5	5	100%
10. Reducing to Lowest Terms:	4.5	4.7	5	4.7	100%
11. Finding the Lowest Common Denominator:	4	4.3	5	4.4	67%
12. Basic Operations with Common Fractions and Mixed Fractions:	4	4.3	5	4.4	67%
Decimal Fractions					
1. Reading and Writing:	5	5	5	5	100%
2. Rounding Off:	5	4.7	5	4.9	100%
3. Basic Operations:	5	5	5	5	100%
4. Table of Decimal Equivalents:	4	3.7	5	4.2	50%
5. Fraction to Decimal Conversion:	4.5	4.7	5	4.7	100%
6. Decimal to Fraction Conversion:	4.5	4.3	5	4.6	100%
Scientific Notation					
1. The Concept of Powers of 10:	5	5	5	5	100%
2. Significant Figures:	5	5	5	5	100%
3. Rounding Off:	5	5	5	5	100%
4. Basic Operations Using Powers of 10:	5	5	5	5	100%
5. Raising to Powers Using Powers of 10:	5	5	5	5	100%
6. Extracting Square Roots Using Powers of 10:	5	5	5	5	100%
Squares and Square Roots					
1. Using the Table of Square Roots:	4	4	5	4.3	60%

	μ V	μ M	μ A	μtot	%4-5
2. Finding the Square of a Number:	5	4.7	5	4.9	100%
3. Finding the Square Root of a Number:	5	4.7	5	4.9	100%
4. Basic Operations of Radicals:	4.5	4	5	4.5	83%
5. The Radical Sign and Index of Roots:	5	4.3	5	4.8	83%
6. The Pythagorean Theorem:	5	4.3	5	4.8	83%
Percentage					
1. Definition:	5	5	5	5	100%
2. Converting Fractions to Percent:	5	5	5	5	100%
3. Converting Decimals to Percent:	5	4.3	5	4.8	83%
4. Converting Percents to Decimals:	5	4.3	5	4.8	83%
5. Finding the Percentage Given the Base and Rate:	5	4	5	4.7	83%
6. Finding the Rate Given the Percentage and Base:	5	4	5	4.7	83%
7. Finding the Base Given the Rate and Percentage:	5	4	5	4.7	83%
8. Solving Problems:	5	4	5	4.7	83%
Ratio and Proportion					
1. Definition of Ratio:	5	5	5	5	100%
2. Reduction of Ratio to Lowest Terms:	4.5	4.7	5	4.7	100%
3. Definition of Proportion:	5	5	5	5	100%
4. Finding the Unknown Term in a Proportion:	5	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
5. Application of Proportion:	4	4.3	5	4.4	67%
6. The English System of Weights and Measures:	4	3	5	4	50%
7. The Metric of Weights and Measures:	5	5	5	5	100%
8. English-Metric Conversion:	5	3	5	4.3	80%
9. The Binary Number System:	5	5	5	5	100%
10. The Computer Number System:	4	5	5	4.7	83%
11. Computing Like a Computer:	4.5	5	5	4.8	83%
12. Binary to Decimal Conversion:	5	5	5	5	100%
13. Decimal to Binary Conversion:	5	5	5	5	100%
14. Basic Operations of Binary Numbers:	5	5	5	5	100%
15. Multiplying and Dividing Measurements:	5	5	5	5	100%
Instruments of Measuring					
1. Linear Scales:	4.5	5	5	4.8	100%
2. The Gauge Number of Wire:	5	4.7	4	4.6	100%
3. Logarithmic Scales:	5	5	5	5	100%
4. Angular Scales:	3.6	3.7	5	4.1	67%
5. Uniform Scales:	3.4	3.7	5	4	67%
Area					
1. Area as a Measure of Surface Space:	4.2	4	5	4.4	78%

	μ V	μ M	μ A	μtot	%4-5
2. Area Calculations of a Circle (Diameter, Circumference, Arcs, Sectors Chords and Tangents):	4.2	4	5	4.4	78%
3. The Properties of π:	4.2	4	5	4.4	78%
Time					
1. Units of Time:	5	5	5	5	100%
2. Distance in Relation to Time:	5	4.3	5	4.8	83%
3. Frequency:	5	5	5	5	100%
4. Period:	5	5	5	5	100%
Geometry					
1. Straight Line & Circle:	3.6	4	5	4.2	67%
2. Translation of Axis:	3.6	4	5	4.2	67%
3. Polar Coordinates:	4.2	4	5	4.4	50%
BASIC ALGEBRA					
General Numbers					
1. Properties of Real Numbers:	4	4	5	4.3	67%
2. Substitution of Variables into Algebraic Expressions:	5	5	5	5	100%
3. Translation of Word Expressions into Algebraic Expressions:	5	5	5	5	100%
4. Order of Operations:	5	5	5	5	100%
5. Evaluating Expressions Containing Variables:	5	5	5	5	100%
6. Basic Operations Involving Variables:	5	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
7. Axioms:	5	5	5	5	100%
8. Constants and Variables:	5	5	5	5	100%
9. Positive and Negative Numbers:	5	5	5	5	100%
10. Powers and Exponents:	5	5	5	5	100%
11. Formulas:	5	5	5	5	100%
12. Ratio and Proportion:	5	5	5	5	100%
Arithmetic Operations with Algebraic Expressions					
1. Basic Operations with Algebraic Numbers:	5	5	5	5	100%
2. Operations with Signs of Grouping:	5	5	5	5	100%
3. The Distributive Law:	4	4.7	5	4.6	100%
4. Equations Involving Parentheses:	5	5	5	5	100%
5. Equations as Symbolic Sentences:	5	5	5	5	100%
6. Removing Parentheses Used to Indicate Multiplication:	4.5	4.7	5	4.7	100%
7. First Degree Equations with Signed Numbers:	5	5	5	5	100%
8. Adding and Subtracting Like Terms:	5	5	5	5	100%
9. Adding and Subtracting Polynomials:	5	5	5	5	100%
10. Multiplication and Division of Monomials and Polynomials:	5	4.3	5	4.8	83%
11. Inserting Signs of Grouping:	5	4.3	5	4.8	83%
12. Rules of Exponents:	5	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
13. The Square of a Binomial Sum or Difference:	3.5	4.3	5	4.3	67%
14. The Product of a Binomial Sum or Difference:	3.5	4.3	5	4.3	67%
15. Factoring the Difference of Two Squares:	4	4	5	4.3	67%
16. Factoring Out the Common Factors:	5	4.7	5	4.9	100%
17. Factoring Polynomials:	4.5	4.3	5	4.6	100%
18. Multiplication by a Common Factor:	5	5	5	5	100%
19. Special Products or Expansions:	4	3.3	5	4.3	56%
20. Extracting Square Roots:	5	4.7	5	4.9	100%
21. Factoring Monomials from Polynomials:	4.5	4	5	4.5	83%
22. Symbols of Grouping:	5	4.3	5	4.8	83%
23. Squaring Binomials:	5	4.5	5	4.8	100%
Algebraic Fractions					
1. Properties of Algebraic Fractions:	4.5	4.3	5	4.6	100%
2. Graphic Representation of Algebraic Fractions:	4.2	3.3	5	4.2	78%
3. Equations Involving Fractions:	5	4.7	5	4.9	100%
4. Basic Operations with Algebraic Fractions:	5	4.7	5	4.9	100%
5. Special Products:	3.8	3.3	5	4	78%
6. The Fractional Exponents:	5	3.3	5	4.4	83%
7. Factoring Fractions:	4	3.3	5	4.1	78%
8. Finding the Least Common Denominator:	4	3.3	5	4.1	78%

	μ V	μ M	μ A	μtot	%4-5
9. Reducing to Lower Terms:	4	3.3	5	4.1	78%
10. Complex Fractions:	4	3.3	5	4.1	78%
11. Improper Fractions and Mixed Expressions:	3.8	3.3	5	4	67%
12. Compound Fractions:	3.6	3.3	5	4	67%
13. Quotients of Polynomials:	3.6	3.3	5	4	67%
Equations					
1. First Degree Equations in One and Two Variables:	4.2	3.7	4	4	78%
2. Literal Equation Formulas:	4.5	4.7	4	4.4	100%
3. Simplification of Equations:	5	5	4	4.7	100%
4. Solution of Equations in Variables:	4.5	5	4	4.5	100%
5. Analysis of a Formula:	4.2	3.7	4	4	78%
6. Transposition of Terms:	4.2	3.7	4	4	78%
7. Transposition of Factors:	4.2	3.7	4	4	78%
8. Translating Problems into Equations:	5	5	4	4.7	100%
9. Operations with Dimensional Units:	4.2	3.7	5	4.3	78%
10. Solving Systems of Equations of Two and Three Unknowns Algebraically:	4.2	3.7	4	4	78%
11. Dimensional Units and Conversion:	4.2	3.7	5	4.3	78%
12. Equations Containing Fractions and Decimals:	4.2	3.7	5	4.3	78%
13. Degree of an Equation:	4.2	3.7	4	4	78%
14. Solving Linear Equations:	5	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
15. Algebraic Solutions to Variables:	5	5	5	5	100%
16. Equations with Unknowns in the Denominators:	5	5	5	5	100%
17. Radical Equations:	4.2	3.7	5	4.3	78%
18. Proportions:	5	5	5	5	100%
19. Proportionality:	4.2	3.7	5	4.3	78%
Linear Graphs					
1. Coordinate Notation:	5	5	5	5	100%
2. Plotting Points on Graphs:	5	5	5	5	100%
3. Graphs of Ordered Pairs:	5	5	3	4.3	83%
4. Displaying Data with Graphs:	5	5	5	5	100%
5. Solving Problems by Means of Graphs:	5	5	3	4.3	83%
6. Graphs of Linear Equations:	5	5	5	5	100%
7. Solve by Comparison of Equalities:	4.5	4.7	3	4.1	83%
8. Solve by Addition & Subtraction:	4.5	4.7	3	4.1	83%
9. Applications of Simultaneous Linear Equations:	4.5	4.7	3	4.1	83%
Quadratic Equations					
1. Solution of $ax^2+bx+c=0$:	4.5	4.3	5	4.6	100%
Exponential Notation					
1. Laws of Basic Operations with Exponents:	5	5	5	5	100%
2. The Law of Zero Exponents:	5	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
3. The Law of Exponents in Raising to Powers:	5	5	5	5	100%
4. The Law of Exponents in Extracting Roots:	5	5	5	5	100%
5. The Law of Exponents Based on 10:	5	5	5	5	100%
6. Power Variations:	2.5	5	5	4.2	83%
7. j-Operator:	5	5	5	5	100%
8. The One Operator:	2.5	5	5	4.2	83%
9. Complex Numbers:	5	5	5	5	100%
10. Complex Equations:	3.5	3.7	5	4.1	67%
11. Squares of Numbers:	5	5	5	5	100%
Radicals					
1. Irrational Numbers:	5	4.7	3	4.2	83%
2. The Square Root of Positive Numbers:	4.6	4.7	5	4.8	100%
3. Equations with Radicals:	3.6	3.3	5	4	67%
Logarithmic Notation					
1. Definition of Logarithms:	4.5	4.7	5	4.7	100%
2. Properties of Logarithms:	4.5	4.7	5	4.7	100%
3. Characteristics:	4.5	4.7	5	4.7	100%
4. Mantissa:	4.5	4.7	5	4.7	100%
5. Determining Logs From a Table:	4.5	4.7	5	4.7	100%
6. Finding Powers Using Logarithms:	5	5	5	5	100%
7. Finding Roots Using Logarithms:	5	5	5	5	100%
8. Interpolation Using Logarithms:	4.5	4.7	3	4.1	83%

	µ V	µ M	µ A	µtot	%4-5
9. Given n, Find Log n:	4.5	5	3	4.2	83%
10. Given Log n, Find n:	4.5	5	3	4.2	83%
11. Graphs and Natural Logarithms:	3.6	3.7	5	4.1	67%
12. Logarithms to the Base E:	4.5	4.7	5	4.7	100%
13. Time Constant Curves:	4	3.7	5	4.2	67%

Sets and Logic

1. Set Notation:	3.6	3.3	5	4	67%
2. Developing Symbols:	3.8	3.3	5	4	67%
3. Electric Switch Diagrams:	5	4.7	5	4.9	100%
4. Venn Diagrams:	5	3.3	5	4.4	83%
5. Truth Tables:	5	4.7	5	4.9	100%
6. Tautologies:	4.5	3.3	5	4.3	83%

Work and Power

1. Work and Power:	4.6	4.7	5	4.8	100%
2. Electrical Power-Watt:	5	5	5	5	100%

ADVANCED ALGEBRA AND ANALYTIC GEOMETRY

Exponents

1. The meaning of Powers Whose Exponents are Either "1" or "0":	5	3	4	4	60%
2. The Meaning of Powers with Negative Whole-Number Exponents.	5	5	4	4.7	100%
3. The Reciprocals of Powers and Converting Divisions to Multiplication:	5	5	3	4.3	80%
4. Converting Multiplications of Powers to Divisions:	5	5	3	4.3	83%

	µ V	µ M	µ A	µtot	%4-5
5. Equivalent Forms of Fractions Containing Powers:	5	5	3	4.3	83%
6. Powers of Fractions:	5	3.7	4	4.2	83%
7. Powers of Powers:	5	3	4	4	80%
8. Adding and Subtracting Letter-Terms which Contain Powers:	5	5	4	4.7	100%
9. A Review of Powers and the Laws of Exponents:	5	5	4	4.7	100%
10. The Roots of Multiplication and Division:	5	3.3	4	4.1	80%
11. The Roots of Powers:	5	3.3	4	4.1	80%
12. Simplifying Radicals by Factoring Out Perfect Powers:	5	3.3	4	4.1	80%
13. Raising Radicals to Powers:	5	4.7	4	4.6	100%
14. The Meaning of Powers with Fraction Exponents:	5	4.7	4	4.6	100%
15. Fractional Exponents and the Laws of Exponents and Radical Powers and Decimal Exponents:	5	3.3	4	4.1	80%
16. The Meaning of Roots and Radicals:	5	4.7	4	4.6	100%
Linear Graphs and Slope					
1. Identifying Linear Equations in "x" and "y":	5	4.7	4	4.6	100%
2. The Intercepts of Linear Exponents:	5	4.7	3	4.2	80%
3. Graphing Linear Equations by Plotting Intercepts:	5	4.7	4	4.6	100%
4. a Graphical Representation of Changes in "y" Corresponding to Increases in "x":	5	4.7	4	4.6	100%

	μ V	μ M	μ A	μtot	%4-5
5. The Slope of a Line Expressed as a Signed Number:	5	3.3	4	4.1	80%
6. The Slope of a Line as a Ratio of Various Pairs of Changes in "x" and "y":	5	4.3	4	4.4	100%
7. Slope Intercept of Linear Equations:	5	4.3	3	4.1	80%
8. Determining the Slope of a Line Through Two Known Points:	5	4.3	3	4.1	80%
Common and Natural Logarithms					
1. The Power-of-Ten Form of Numbers Greater Than 1:	4	4.7	4	4.2	100%
2. Common Logarithms of Numbers Greater Than 1:	4	5	4	4.3	100%
3. Common Logarithms of Numbers Between 0 and 1:	4	5	5	4.7	100%
4. Logarithmic Scales:	4	5	5	4.7	100%
5. Semi-Log and Log-Log Graphs:	5	5	5	5	100%
6. The Number "e" as a Base:	5	5	5	5	100%
7. Evaluations with Formulas Which Contain "e":	5	5	5	5	100%
8. Natural (Base "e") Logarithms:	5	5	5	5	100%
9. Evaluating "Ln" Formulas:	5	5	3	4.3	80%
10. Evaluating "Ln" Formulas When a Conversion of Power-of-"e" form is required:	5	5	3	4.3	80%
Logarithms and Exponential Laws and Formulas					
1. The Laws of Common Logarithms:	4	5	4	4.3	100%

	μ V	μ M	μ A	μtot	%4-5
2. The Laws of Natural Logarithms:	4	5	4	4.3	60%
TRIGONOMETRY					
Right Triangles and Trigonometric Ratios					
1. Relationship Between θ and the Coordinates of P:	5	5	3	4.3	83%
2. Variations in Right Triangle Problems:	5	5	5	5	100%
3. Linear Velocity:	5	4	4	4.3	83%
4. Right Triangles and Angle-Sum Principle:	4.5	5	5	4.8	100%
5. Right Triangles and the Pythagorean Theorem:	5	5	5	5	100%
6. Scaling Method and Right Triangles:	4	4.7	5	4.6	100%
7. Definition of the Tangent Ratio:	5	5	3	4.3	83%
8. The Sine Ratio-A Partial Table:	4	3.7	5	4.2	50%
9. The Cosine Ratio-A Partial Table:	4	3.7	5	4.2	50%
10. Finding the Unknown Sides in Right Triangles:	5	4.7	5	4.9	100%
11. Finding the Unknown Angles in Right Triangles:	5	4.7	4	4.9	100%
Vectors & Phasors					
1. Definition of Vectors and Phasors:	5	5	5	5	100%
2. Vectors on the Rectangular Coordinate System:	5	5	3	4.3	83%
3. Basic Operations with Phasors:	5	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
4. The Trig Ratios of Reference Angles:	5	5	3	4.3	83%
5. Finding the Length and Director of a Slanted Vector:	5	5	5	5	100%
6. Addition of Vectors by the Component Method:	5	5	5	5	100%
7. Finding the Length and Director of a Resultant:	5	5	5	5	100%
8. Adding Vectors which Lie on the Coordinate Axes:	5	5	3	4.3	83%
9. Finding the Vector-Addend:	5	5	3	4.3	83%
10. Adding More Than Two Vectors:	5	5	5	5	100%
11. Vector Opposites, the Zero-Vector, and the State of Equilibrium:	4.5	5	5	4.8	83%
12. Equilibrants and the State of Equilibrium:	4.5	5	5	4.8	83%
Trigonometric Ratios of Standard-Position Angles					
1. Principle Angles:	5	4.7	5	4.9	100%
2. Standard-Position Angles Between 0° and 360°:	3.5	3.7	5	4.1	67%
3. Reference Angles of Standard-Position Angles Between 0° and 360°:	4.5	4.7	3	4.1	83%
4. Finding the Function of any Angle:	4.5	4.7	3	4.1	83%
5. Finding Standard-Position Angles Corresponding to known Ratios:	4.5	4.7	3	4.1	83%
6. Finding the Angles Given the Function:	3.6	3.7	5	4.1	67%
7. Trigonometric Ratios of 0°, 90°, 180°, 270°, and 360°:	5	4.7	3	4.2	83%

	μ V	μ M	μ A	μtot	%4-5
8. The Basic Cycle of the Sine-Wave Graph, Graphing Trig. Functions:	4.5	4.7	5	4.7	100%
9. Vector-Addition and Standard Position Angles:	4.5	4.7	3	4.1	83%
10. Related Angles:	5	4.7	3	4.2	83%
11. Other Cycles of the Sine-Wave Graph:	5	4.7	5	4.9	100%
12. Functions of Angles in Quadrants I, II, III, and IV:	4	4	5	4.3	67%
13. Definition of a "Degree" in Angle Measurement:	4.5	4.7	4	4.4	100%
14. Definition of a Radian in Angle Measurement:	4.6	4.7	3	4.1	83%
15. The Relationship Between Degrees and Radians:	4.6	4.7	3	4.1	83%
16. Radians Expressed in Terms of "π":	4.8	4.7	3	4.2	83%
17. Trig Ratios of Angles Expressed in Radians:	4.8	4.7	3	4.2	83%
Sine Wave Analysis					
1. Fundamental Sine Waves:	5	5	5	5	100%
2. Non-Fundamental Sine Waves:	5	5	5	5	100%
3. Phase Differences Between Non-Fundamental and Fundamental Sine Waves:	5	5	5	5	100%
4. The Concepts of "Lead" and "Lag":	5	5	5	5	100%
5. The Broader Interpretation of Phase Difference and the Concepts of "Lead" "Lag":	5	5	5	5	100%
6. Sine Wave Harmonics:	5	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
7. Sketching Fundamental Sine Waves:	5	5	5	5	100%
8. Sketching Non-Fundamental Sine Waves:	5	5	5	5	100%
9. Sketching Harmonics of Fundamental Sine Waves:	5	5	5	5	100%
10. Sine Waves with Negative Amplitudes:	5	5	5	5	100%
11. Cosine Waves:	5	5	5	5	100%
Complex Numbers					
1. j-Operator:	5	4.7	5	4.9	100%
2. Real and Imaginary Numbers:	5	4.7	5	4.9	100%
3. Complex Numbers and Their Vector Representation:	5	4.7	4	4.6	100%
4. Vector-Addition by Means of Complex Numbers:	5	4.7	4	4.9	100%
5. Vector-Subtraction by Means of Complex Numbers:	5	4.7	4	4.9	100%
Periodic Functions					
1. Circular Functions:	3.2	4	5	4.1	56%
2. Graphing of y=sin θ:	3.2	4	5	4.1	56%
3. Graph of y=cos θ:	3.2	4	5	4.1	56%
4. Trig Functions; f(x)=Sin(x); g(x)=Cos(x):	3.8	4.7	4	4.2	83%
5. Amplitude:	4.6	4.7	5	4.8	89%
6. Frequency:	4.6	4.7	5	4.8	89%
7. Addition and Sinusoidal Waves:	4.2	4.7	5	4.6	78%

	μ V	μ M	μ A	μtot	%4-5
THE D. C. CIRCUIT					
1. Difference of Potential and Electric Current:	5	5	5	5	100%
2. E.M.F. and Energy Considerations:	5	5	5	5	100%
3. Ohm's Law:	5	5	5	5	100%
4. Voltage Drops:	5	5	5	5	100%
5. Energy and Power Dissipated in a Circuit:	5	5	5	5	100%
6. Series and Parallel Circuits; Equivalent Resistance:	5	5	5	5	100%
7. Direction of Current:	5	5	5	5	100%
8. Kirchhoff's Rules:	5	5	5	5	100%
9. Battery E.M.F. and Terminal Voltage -- Internal Resistance:	5	5	5	5	100%
10. Wheatstone Bridge:	5	5	5	5	100%
FUNDAMENTAL ELEMENTS IN AN A.C. CIRCUIT					
1. Capacitance:	5	5	5	5	100%
2. Capacitors in Series:	5	5	5	5	100%
3. Capacitors in Parallel:	5	5	5	5	100%
4. Energy Required to Charge A Capacitor:	5	5	5	5	100%
5. Time for Charging and Discharging a Capacitor; the Timer Constant:	5	5	5	5	100%
6. Inductance:	5	5	5	5	100%
7. Time Constant of an L-R Circuit:	5	5	5	5	100%
8. Phase Relation in A.C. Circuits:	5	5	5	5	100%

	μ V	μ M	μ A	μtot	%4-5
9. The Series A.C. Circuits; Phase Diagram:	5	5	5	5	100%
10. Reactance and Impedance:	5	5	5	5	100%
11. Resonance:	5	5	5	5	100%
12. Power and Power Factor:	5	5	5	5	100%

APPENDIX D

Mini-Module Packets for Selected Competencies

- Least Common Denominator
- Addition of Fractions
- Subtraction of Fractions
- Multiplication of Fractions
- Division of Fractions

It is a well established fact that not all students learn at the same rates nor do they learn equally well by the same methods of approach to the subject matter. Not to replace classroom instruction, but rather as a supplement to it, this research project has sought to develop materials that might be used as an aid to the development of alternative teaching aids related to mathematics in a variety of vocational areas.

In the pages that follow are five programmed instruction units which cover related concepts of importance to all the vocational areas considered by this project. Each of the units approaches its concept from a purely mathematical standpoint. It should be pointed out that once a student has mastered the basic concept, additional units could be made available which would present applications of the concept to the student's particular vocational field.

Following the programmed instruction units are transcripts of audio tapes that cover the same related concepts. Each transcript is accompanied by a short-form workbook which allows the student to follow the text, do scratch work, and write in problem answers as he listens to the tape.

The programmed instruction books and audio tapes give the student a choice as to the best or easiest way for him to study a topic. Both the modes of instruction have been pilot tested and have given very encouraging results. Based on comparisons of pre and post-tests, only about 10% of those tested showed no improvement in performance level.

Student feedback was extremely favorable especially with regard to the audio tapes. Minority students for whom English is a second language, and other students who had difficulty with reading textbooks indicated they gained far more from the tapes than they did from trying to struggle through a standard math text.

Least Common Denominator Module

GENERAL INSTRUCTIONS

1. Before beginning this module, take the pre-test assessment to determine your knowledge for Least Common Denominators and their prerequisites.
2. Select one of the two instructional modes--cassette tape or programmed instruction--and complete the study module.
3. Take the post-test to determine your mastery of the Least Common Denominator Concept.

PREREQUISITES

The following prerequisites should have been mastered before undertaking this module.

1. Least Common Multiple
2. Prime Number
3. Factor of a Number
4. Factoring a Number into Prime Factors.

PRE AND POST-TEST PERFORMANCE SPECIFICATIONS

Given two different fractions you will find the least common denominator and record your answer on the assessment instrument. There will be five examples on the instrument. The acceptable performance level will be 80% correct responese.

INSTRUCTIONAL MODES

Choose one of two different instructional modes. Instructional mode number 1 consists of a cassette tape on how to find the least common denominator for a series of fractions. Instructional mode number 2 consists of a series of programmed instructional frames on how to find the least common denominator of a set of fractions.

PROGRAMMED INSTRUCTION
for
LEAST COMMON DENOMINATORS

Before beginning this module, it is necessary to take the pre-test on Least Common Denominators. Programmed instruction is used as a workbook. Each frame contains one or more blanks in which you should write the numbers you feel best complete the statement in the frame.

3 x 3 = ________

9

It is necessary to use some device (a 5" x 8" index card is good) to cover the answer which appears directly below the frame that you are working. It must be pointed out that for maximum benefit, you must not look at the answer before you record your responses in the frame.

Good Luck!

LEAST COMMON DENOMINATORS

1. In the expression (2/3 + 4/5 + 9/10), the numbers 3, 5, 10 are the _________ of the terms in which they appear?

---

denominators

---

2. The smallest number which each denominator in (2/3 + 4/5 + 9/10), divides is _________?

---

30 (not 50, 150, ...)

---

3. 30 is the smallest number that is divisible by 3, 5 and 10. Thus, the Least Common Multiple (LCM) of 3, 5 and 10 is _________?

30

---

4. 32 is the _________ of 8, 12 and 32?

Least Common Multiple
(or LCM)

---

5. What is the LCM of 3, 4, and 5? _________

60

---

6. Some whole numbers may be "factored", or written as the product of smalled whole numbers; e.g., 14 = 2 x 7. 21 = ________ x ________?

---

3 x 7

---

7. 24 may be factored as: 24 = 12 x ________

= 4 x ________

= 3 x ________

= 3 x 2 x ________

---

24 = 12 x 2

= 4 x 6

= 3 x 8

= 3 x 2 x 4

---

8. A number is factored completely when it is written as the product of prime numbers. A prime number is any whole number (other than 1) that is divisible only by itself and by 1. Is 23 a prime number? ________

Yes

---

9. 24 is completely factored as 2 x 2 x ________ x ________.

2, 3

10. 24 = 2 x 2 x 2 x 3

To factor a number completely, start with the smallest prime number, 2, and ask if it is a factor of the number.

To factor 36 completely, such as: 36 = 2 x ________.

18

11. 2 is not a factor of 9.

Try the next larger prime number and ask if 3 is a factor of 9. ________

36 = 2 x 18

36 = 2 x 2 x 9

36 = 2 x 2 x 3 x ______

Yes, 3 is a factor of 9

36 = 2 x 2 x 3 x 3

12. 36 has now been factored to its prime factors. 36 = 2 x 2 x 3 x 3.

Can any of these factors be factored? ________

No, by the definition of prime numbers.

13. To factor 294 completely, ask if 2 is a factor, then 3, then 5, etc....

Always progress in this order; never just try factors at random.

The smallest prime number that is a factor of 294 is ________?

2

14. 294 = 2 x ________

147

---

15. Examine the final factor, 147 and decide whether it contains 3 as a factor.

294 = 2 x 147
294 = 2 x 3 x ________

49

---

16. Again examine the final factor, 49. 3 is obviously not a factor of 49.
What factor should be tried next? ________

294 = 2 x 147
294 = 2 x 3 x 49
294 = 2 x 3 x _____ x _____

The next factor is 7.
7, 7

---

17. 294 = 2 x 3 x 7 x 7
294 is completely factored as shown here. Only the systematic approach just reviewed will ensure finding every prime factor.
Factor completely: 56 = ________

---

56 = 2 x 2 x 2 x 7

---

18. Factor completely: 45 = __________

36 = __________

---

45 = 3 x 3 x 9

36 = 2 x 2 x 3 x 3

---

19. Factor completely: 60 = __________

63 = __________

---

60 = 2 x 2 x 3 x 5

63 = 3 x 3 x 7

---

20. Complete factoring may be used to find the LCM of any series of numbers; e.g., 10 and 15.

10 = __________ x __________

15 = __________ x __________

---

10 = 2 x 5

15 = 3 x 5

---

21. 10 = 2 x 5

15 = 3 x 5

The LCM must contain all the prime factors of 10 and 15 but without duplication of common factors.

The LCM of 10 and 15 must contain the factors __________, __________, and __________.

---

22. 2, 3, 5 (Notice that 5 appears as a factor only once.)

---

23. 10 = 2 x 5

15 = 3 x 5

The factors in the LCM of 10 and 15 are 2, 3 and 5.

LCM of 10 and 15 = __________ x __________ x __________ = __________?

---

2 x 3 x 5 = 30

---

24. Factor completely: 16 = __________

40 = __________

---

16 = 2 x 2 x 2 x 2

40 = 2 x 2 x 2 x 5

---

25. 16 = 2 x 2 x 2 x 2

40 = 2 x 2 x 2 x 5

The LCM must contain each factor the maximum number of times it is used in either 16 or 40. The factors of 16 include __________ (How many?) 2's?

The LCM must therefore contain __________ (How many?) 2's?

---

four

four

---

26. 16 = 2 x 2 x 2 x 2

40 = 2 x 2 x 2 x 5

We know that the LCM must contain four 2's since there are four 2's in 16. It need contain only one __________, since this factor appears only once in 40?

5

---

27. 16 = 2 x 2 x 2 x 2
40 = 2 x 2 x 2 x 5
Thus, the LCM must contain four 2's and one 5. Are there any other factors of either 16 or 40 that must be included in the LCM? __________

no

28. 16 = 2 x 2 x 2 x 2
40 = 2 x 2 x 2 x 5
LCM = __________ x __________ x __________ x __________ x __________ = ______

LCM = 2 x 2 x 2 x 2 x 5 = 80

29. Factor completely: 66 = __________
55 = __________
LCM of 66 and 55 = __________

66 = 2 x 3 x 11
55 = 5 x 11
LCM = 2 x 3 x 5 x 11

30. 66 = 2 x 3 x 11
66 = 5 x 11
LCM = 2 x 3 x 5 x 11

The only factor common to 66 and 55 is __________.

11

31. Factor the following and find the L.C.M.

$66 = 2 \times 3 \times 11$ 28 = ________

$\underline{55 = 5 \times 11}$ 35 = ________

Thus, LCM = 2 x 3 x 5 x 11 LXM of 28 and 35 = ________

$$28 = 2 \times 2 \times 7$$
$$\underline{35 = 5 \times 7}$$
$$\text{LCM} = 2 \times 2 \times 5 \times 7$$

---

32. Factor the following and find the L.C.M.

62 = ________

93 = ________

LCM of 62 and 93 = ________

$$62 = 2 \times 31$$
$$\underline{93 = 3 \times 31}$$
$$\text{LCM} = 2 \times 3 \times 31$$

---

33. The first step in adding and subtracting fractions is to determine the Least Common Denominator for the fractions involved. We will first consider several numerical fractions and then consider the general case of algebraic fractions.

34. The Least Common Denominator of the series of fractions is simply they LCM of the ________ of the fractions.

denominators

35. To find the Least Common Denominator (LCD) of 4/27 and 6/45, first determine the ________ of 27 and 45.

LCM

36. LCM of 27 and 45 = ____________ = ________
(Factored form)
LCD of 4/27 and 6/45 = ________

3 x 3 x 3 x 5 = 135
135

37. Find the Least Common Denominator (LCD) of 1/6, 3/8, and 5/12.
LCD = ________?

2 x 2 x 2 x 3 or 24

38. The only factor common to the denominator of 87/52, 19/39, and 25/78 is ________.

13

39. Find the LCD of 87/52, 19/39 and 25/78.

LCD = __________?

2 x 2 x 3 x 13 or 156

40. Since the denominators of 1/3, 1/5 and 1/7 cannot be factored, each must be a factor in the LCD. The factors in the LCD are __________, __________, and __________.

3, 5, 7

41. To find the LCD of 3/8, 5/9 and 7/12, first find the factors of each denominator.

8 = 2 x 2 x 2 14 = __________ 28 = __________

9 = 3 x 3
12 = 2 x 2 x 3

42. 3/8, 5/9, 7/2 8 = 2 x 2 x 2 9 = 3 x 3 12 = 2 x 2 x 3

The maximum number of times 2 is a factor in any of the above denominators __________.

The maximum number of times 3 is a factor in any of the above denominators __________.

3
2

43. 8 = 2 x 2 x 2
9 = 3 x 3
12 = 2 x 2 x 3

The LCM of the denominator 8, 9, 12 is ________.

---

2 x 2 x 2 x 3 x 3 x 3 = 216

---

44. The LCD for 1/2, 1/6 and 1/9 is ________.

18

---

45. The LCD for 3/14, 5/28 and 3/35 is ________.

140

---

46. The LCD for 3/17, 2/34 and 1/68 is ________.

68

---

47. The LCD for 1/10, 4/15 and 7/25 is ________.

150

---

48. 5/36, 29/42, 1/63

Factor (prime factors)

36 = ________

42 = ________

63 = ________

---

36 = 2 x 2 x 3 x 3

42 = 2 x 3 x 7

63 = 3 x 3 x 7

---

49. 5/36, 29/42, 1/63

36 = 2 x 2 x 3 x 3

42 = 2 x 3 x 7

63 = 3 x 3 x 7

LCD = ________

LCD = 2 x 2 x 3 x 3 x 7 = 252

---

50. You have now reviewed the most efficient way to determine the Least Common Denominator for any series of numerical fractions.

You have also calculated a sufficient number and variety of LCD's to become quite proficient in the application of the technique reviewed.

STOP

Tape on
Least Common Denominators

This self study module will teach you about the Least Common Denominator of fractions.

Frame number one gives the expression, 3/7. It is composed of two parts, a numerator and denominator. The numerator occupies the top position, and is 3. The denominator occupies the bottom position, and it is 7.

Frame number two gives three fractional expressions. Identify the numerator, and the denominator, and write your responses in the appropriate blanks.

The answers should be: the numerators are 3, 4, and 5; and the denominators are 8, 13, and 9.

Frame number three gives the fractional expression, 3/5 + 3/25. The smallest number which divides each denominator is 5.

Frame number four gives the fractional expression, 1/2 + 1/4. What is the smallest number which divides each denominator?

The answer is 2.

Frame 5 gives the fractional expression 3/13 + 7/26 + 11/39. What is the smallest number that the denominators will divide?

Your answer should be 78.

The smallest number that is divisible by 8, 16, and 32, is 32. Thus, the Least Common Multiple, also called LCM of 8, 16, and 32, is 32. Consider the numbers 6, 16, and 24, as listed in frame 6. The Least Common Multiple is 48.

In frame seven, what is the LCM (Least Common Multiple) of 3, 4, and 5?

The answer is 60.

In the next frame, what is the LCM of 13, 26, and 39?

The answer is 78.

Some whole numbers may be written as the product of smaller whole numbers. For example, look at frame 9. Notice in frame 9, that 16 can be written several ways. Expression c shows 16 written as a product involving 2 x 2 x 2 x 2. This is what is called the complete factored form.

In frame 10, write in the missing numbers.

The answers are: for a) 2; for b) 6 or 2 x 3; for c) 8 or 2 x 2 x 2; for d) 4 or 2 x 2.

A number is factored completely when it is written as a product of prime numbers. A prime number is any whole number other than one, that is divisible by itself and one.

Least Common Denominators

In frame 11, there are the numbers, 3, 7, 8, 5 and 12. It is seen that 3, 7 and 5 are prime numbers and that 8 and 12 can be written as products with more than one and themselves.

In frame 12, identify the prime numbers.

The answers are: a, b, and c are prime numbers; d is not a prime number.

In frame 13, complete the factorization of 24.

Your answer should be 2.

In frame 14, complete the factorization of 18.

The answer is 3 x 3 x 2.

In frame 15, factor the number 36.

The answer is 3 x 2 x 3 x 2 or any other order of those numbers such as 3 x 3 x 2 x 2.

In frame 16, factor the number 72.

Your answer should read 3 x 3 x 2 x 2 x 2.

In frame number 17, factor the numbers 35, 16, 60 and 63.

Here are the answers: for a) 35 = 5 x 7; b) 16 = 2 x 2 x 2 x 2; c) 60 = 2 x 2 x 3 x 5; d) 63 = 3 x 3 x 7.

Complete factoring, for example the factoring of 16 which would equal 2 x 2 x 2 x 2, may be used to find the LCM (Least Common Multiple) of any series of numbers. The following example in frame 18 shows the numbers 10 and 15 completely factored.

The LCM (Least Common Multiple) of numbers 10 and 15 must contain all the prime factors without duplication of common factors. For example, the LCMs of 10 and 15 contain the factors 2, 3, and 5. Notice that 5 appears only once. As it was said before, the LCM contains all the prime factors without duplication.

In frame 19, the numbers 26, and 39 are completely factored. The LCM contains what factors?

The answer is 2, 3, and 13.

In frame 20, factor the numbers 18 and 30 completely.

Here are the correct answers. 18 = 2 x 3 x 3; 30 = 2 x 3 x 5.

The LCM contains each factor the maximum number of times it is used in either 18 or 30. The factors of 18 include how many 3's?

The answer is 2.

In the next question, the LCM must contain how many 3's?

Again the answer is 2.

Least Common Denominators

The LCM in frame 22 contains two 3s, since there two 3s in 18. It need contain only one 2 since this factor appears only once in 18, and once in 30. Thus, the LCM must contain two 3s and one 2. What other factor of 18 and 30 must be included in the LCM? See frame 22.

5 is the answer.

In frame 23, the LCM of 18 and 30 is found. First factor 18 and 30 completely, then identify the factors and multiply to find the LCM.

Here is the answer: The LCM equals 2 x 3 x 3 x 5, which also equals 90.

In frame 24, find the LCM of 66 and 55. First factor 66 and 55 completely, then identify the factors and multiply to find the LCM.

The answer is: 66 = 2 x 3 x 11; 55 = 5 x 11. The LCM is 2 x 3 x 5 x 11 = 330.

For frame 25, find the LCM of 62 and 93.

Now, for the answers: 62 = 2 x 31; 93 = 3 x 31. The LCM = 2 x 3 x 31 = 186.

In frame 26, find the LCM of the numbers 28 and 21.

The answers are: 28 = 2 x 2 x 7; 21 = 3 x 7. The LCM = 2 x 2 x 3 x 7 = 84.

Now we will discuss the Least Common Denominator. The Least Common Denominator of a series of fractions is simply the LCM (Least Common Multiple) of the denominators of the fractions. To find the Least Common Denominator, also known as the LCD, of the fractions 4/27 and 6/45, first determine the Least Common Multiple of 27 and 45. In the next frame, the LCM of 27 and 45 is what number?

Here is the answer: 27 = 3 x 3 x 3; 45 = 3 x 3 x 5. The LCM = 3 x 3 x 3 x 5 = 135.

In frame 28, what is the Least Common Denominator or LCD of the numbers 27 and 45?

The answer is 135. Note that this answer is the same as the LCM in the preceeding question. Thus, to review: The Least Common Denominator of a series of fractions is simply the Least Common Multiple of the denominators of the fractions.

In frame 29, find the Least Common Denominator of 3/35, 4/55, and 1/77.

Here are the answers: 35 = 5 x 7; 55 = 5 x 11; 77 = 7 x 11. The Least Common Denominator equals 5 x 7 x 11 which, multiplied out, equals 385.

Now, in frame 30, find the LCD of 87/52, 19/39, and 25/78.

The correct answer is: The Least Common Denominator equals 2 x 2 x 3 x 13, which equals 156.

You have now completed the most efficient way of finding the Least Common Denominator. You should now take the post-test on how to find the Least Common Denominators to test your abilities, now that you have studied the procedures.

Name ________________

Workbook for Cassette Tape
on
Least Common Denominators

1.
$\frac{3}{7}$

2.
$\frac{3}{8}$, $\frac{4}{13}$, and $\frac{5}{9}$

Numerators ________

Denominators ________

3.
$\frac{3}{5} + \frac{3}{25}$

4.
$\frac{1}{2} + \frac{1}{4}$

Answer ________

5.
$\frac{3}{13} + \frac{7}{26} + \frac{11}{39}$

Answer ________

6.
6, 16 and 24

7.
3, 4 and 5

Answer _____

6.
13, 26, and 39

Answer ________

9.
a) 16 = 8 x 2
b) 16 = 4 x 2 x 2
c) 16 = 2 x 2 x 2 x 2

10.
a) 24 = 12 x __________
b) 24 = 4 x __________
c) 24 = 3 x __________
d) 24 = 3 x 2 x __________

11.
a) 3 = 3 x 1
b) 7 = 7 x 1
c) 8 = 2 x 2 x 2 x 1
d) 5 = 5 x 1
3) 12 = 3 x 2 x 2 x 1

12.
a) 11 __________
b) 13 __________
c) 29 __________
d) 12 __________

13.
24 = 3 x 2 x 2 x __________

14.
18 = 3 x __________ x __________

15.
36 = __________ x __________ x __________ x __________

16\.

72 = __________

17\.

a) 35 = __________

b) 16 = __________

c) 60 = __________

d) 63 = __________

18\.

10 = 2 x 5

15 = 3 x 5

19\.

26 = 2 x 13 Answer __________

39 = 3 x 13

20\.

18 = __________

30 = __________

21\.

18 includes __________ 3's

LCM includes __________ 3's

22\.

18 = 2 x 3 x 3 Answer __________

30 = 2 x 3 x 5

23\.

18 = 2 x 3 x 3

30 = 2 x 3 x 5

LCM = __________ x __________ x __________ x __________ = ________

24.

66 = ________ x ________ x ________

55 = ________ x ________

LCM = ________ x ________ x ________ x ________

25.

62 = ________

93 = ________

LCM = ________

26.

28 = ________

21 = ________

LCM = ________

27.

27 = ________

45 = ________

LCM = ________

28.

LCD of 27 and 45 = ________

29.

35 = ________

55 = ________

77 = ________

LCD = ________

30.

Find LCD for $\frac{87}{52} + \frac{19}{39} + \frac{25}{78}$ LCD = ________

Addition of Fractions

GENERAL INSTRUCTIONS

1. Before beginning this module, take the pre-test assessment to determine your knowledge for Addition of Fractions and their prerequisites.
2. Select one of the two instructional modes -- cassette tape or programmed instruction -- and complete the study module.
3. Take the post-test to determine your mastery of Addition of Fractions.

PREREQUISITES

The following prerequisites should have been mastered before undertaking this module.

1. Least Common Multiple
2. Factoring a Number into Prime Factors
3. Least Common Denominator

PRE AND POST-TEST PERFORMANCE SPECIFICATIONS

Given two different fractions you will add and record your answer on the assessment instrument. There will be five examples on the instrument. The acceptable performance level will be 80% correct responses.

INSTRUCTIONAL MODES

Choose one of two different instructional modes. Instructional mode number 1 consists of a cassette tape on how to add a series of fractions. Instructional mode number 2 consists of a series of programmed instructional frames on how to add a set of fractions.

PROGRAMMED INSTRUCTION
for
ADDITION OF FRACTIONS

Before beginning this module, it is necessary to take the pre-test on Addition of Fractions. Programmed instruction is used as a workbook. Each frame contains one or more blanks in which you should write the numbers you feel best complete the statement in the frame.

3 x 3 = ________

9

It is necessary to use some device (a 5" x 8" index card is good) to cover the answer which appears directly below the frame that you are working. It must be pointed out that for maximum benefit, you must not look at the answer before you record your responses in the frame.

Good Luck!

ADDITION OF FRACTIONS

1. Programmed texts are used, not read. Each fram contains one or more blanks in which you should write the numbers or symbols you feel best complete the statement in the frame.

2 x 2 = ________

4

---

2. As soon as you have written your answer, but not before, look below and find the correct answer.

2 x 5 - 1 = ________

6

---

3. Compare your answer with the correct answer. If your answer is the same, proceed to the next frame; if your answer is different, study the frame and the answer until you identify your error. Programs develop progressively and you must understand each frame before you proceed to the next.

4 x 2 x 1 = ________

8

---

4. In order for you to derive the maximum benefit from this program, it is essential that you write your answers and check them immediately.

$\frac{6 + 4}{2}$ = ________

5

---

5. Read all instructions carefully and work as rapidly and as accurately as you can. Remember to writ your answers and to check them immediately.

(7 + 3)(2) = (________) (2) = ________

----

(10)(2) = 20

---

6. The programmed review of addition of fractions begins with the next frame. The first Segment of this program consists of 16 frames.

Good Luck!

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7. Two or more numbers multiplied together are termed fractors, and the result of the multiplication is termed their product.

2 x 4 x 5 = 40   Product = ________   Factors = ________

Product = 40   Factors = 2, 4, 5

---

8. 2 x 4 x 5 = 40   Product = 40   Factors = 2, 4, 5

If the factors are divisible only by themselves and by1, they are called prime factors.

Are the factors of 40 listed above all prime factors?

No; 4 is divisible by 2 as well as 4 and 1.

---

9. 3 x 7 x 5 = 105

Are 3, 7, and 5 the prime factors of 105?

Yes

10. The prime factors of 26 are 13 and 2; i.e., 26 = 13 x 2.

Similarly: 12 = 2 x 2 x __________

16 = 2 x 2 x 2 x __________

8 = __________ x __________

12 = 2 x 2 x 3

16 = 2 x 2 x 2 x 2

8 = 2 x 2 x 2

11. Recall that the Least Common Multiple (LCM) of a series of numbers is the (smallest/largest) whole number that is divisible by every member of the series.

Smallest

12. 8 = 2 x 2 x 2

12 = 2 x 2 x 3

16 = 2 x 2 x 2 x 2

LCM of 8, 12, 15 = 2 x 2 x 2 x 2 x __________

3

13. 14 = 2 x 7

21 = 3 x 7

49 = 7 x 7

LCM = _______ x _______ x _______ x _______
(Give prime factors)

2 x 3 x 7 x 7

14. The Least Common Denominator is simply the LCM of the denominators of a series of fractions.

The LCD for 1/6, 3/8, 5/2 is _________.

6 = 2 x 3

8 = 2 x 2 x 2

12 = 2 x 2 x 3

LCD = 2 x 2 x 2 x 3

15. Find the Least Common Denominator (LCD) for the fractions in this expression:

3/10 + 2/5 + 7/9 LCD = _________

10 = 2 x 5

5 = 1 x 5

9 = 3 x 3

LCD = 1 x 2 x 3 x 3 x 5 = 90

16. 5/9 + 1/9 + 2/9 = $\frac{(\qquad) + (\qquad) + (\qquad)}{9}$ = _________

$\frac{(5) + (1) + (2)}{9} = \frac{8}{9}$

17. $9/10 + 5/10 + 3/10 = \frac{(\quad) + (\quad) + (\quad)}{10} = \underline{\qquad}$

$$\frac{(9) + (5) + (2)}{10} = \frac{8}{10}$$

18. In the last two frames, you added fractions that had the same denominators. The numerators of fractions may be added directly only when they share the same denominators. When the denominators of a series of fractions are different, each fractions in the series must first be changed to an equivalent fraction with the same common denominator.

19. To express 3/8 as a fraction whose denominator is 32:

$$\frac{3}{8} = \frac{3}{8} \times \frac{(\quad)}{(\quad)} = \frac{(\quad)}{32}$$

3/8 x 4/4 = 12/32

20. $\frac{5}{14} = \frac{(\quad)}{42}$

5/14 x 3/3 = 15/42

21. To solve 11/35 = ?/210 efficiently, consider the prime factors of the denominators: 35 = 5 x 7 210 = __________

5 x 7 x 6

22. 35 = 5 x 7 11/35 = ?/210

210 = 5 x 6 x 7

________ is the only factor 210 that is not a factor of 35. Thus, 11 and 35 must both be multiplied by ________ to obtain the equivalent fraction ________.

6

6

$\frac{66}{210}$

---

23. Since 14 = 2 x 7 and 42 = 2 x 3 x 7,

$\frac{3}{14} \times \frac{(\quad)}{(\quad)} = \frac{(\quad)}{42}$

3/14 x 3/3 = 9/42

---

24. To solve 15/42 = ?/168

42 = ________ (Give prime factor)

168 = ________ (Give prime factor)

---

42 = 2 x 3 x 7

168 = 2 x 2 x 2 x 3 x 7

---

25. Use prime factors to solve: $\frac{9}{44} = \frac{(\quad)}{220}$

---

44 = 2 x 2 x 11

220 = 2 x 2 x 5 x 11

9/44 x 5/5 = 45/220

26. You have mastered the conversion of a fraction to an equivalent fraction with a different denominator. This technique will now be employed to add fractions.

The next several frames contain a step-by-step consideration of a sequence of operations in several examples.

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27. To perform addition such as 2/14 + 3/48 + 4/70, First determine the LCD by use of prime factors.

14 = __________

49 = __________

70 = __________

LCD = __________

- -

14 = 2 x 7

49 = 7 x 7

70 = 2 x 5 x 7

LCD = 2 x 5 x 7 x 7

= 490

---

28. 2/14 + 3/49 + 3/70 (490 = 2 x 5 x 7 x 7)

$\frac{2}{14} = \frac{70}{490}$ $\frac{3}{49} = \frac{(\_\_\_\_\_\_)}{490}$ $\frac{3}{70} = \frac{(\_\_\_\_\_\_)}{490}$

- -

$\frac{3}{49} \times \left(\frac{2 \times 5}{2 \times 5}\right) = \frac{30}{490}$

$\frac{3}{70} \times \frac{7}{7} = \frac{21}{490}$

---

29. Note that $\frac{2 \times 2 \times 3}{2 \times 3 \times 5}$ may be regrouped as $\frac{2}{2} \times \frac{3}{3} \times \frac{2}{5}$.

Since 2/2 = __________ and 3/3 = __________, the elimination of these factors common to numerator and denominator (will/will not) change the value of the fraction.

- -

1, 1

will not

---

30. The eliminator of factors common to both numerator and denominator of a fraction is termed reducing.

Reduce: $\frac{3 \times 7 \times 11}{4 \times 5 \times 7 \times 11} = (\_\_\_\_) \times (\_\_\_\_) \times \frac{(\quad\quad)}{(\quad\quad)(\quad\quad)} = \_\_\_\_$

$\frac{7}{7} \times \frac{11}{11} \times \frac{3}{4 \times 5} = \frac{3}{4 \times 5}$

---

31. Recall that the factors of 168 are 2 x 2 x 2 x 3 x 7.

Does 87 contain any of these same factors? Which? ________

Yes, 3 (87 = 3 x 29)

---

32. Reduce: $\frac{87}{168} =$ ________

$\frac{87}{168} = \frac{3 \times 29}{2 \times 2 \times 3 \times 7}$

$= \frac{29}{42}$

---

33. To reduce a fraction such as 70/105: (1) express numerator and denominator as prime factors, and (2) reduce where possible.

$\frac{70}{105} = \frac{2 \times 5 \times 7}{3 \times 5 \times 7} =$ ________

$\frac{2}{3}$

34. 3/15 + 1/20 + 3/5 LCD = 5 x 3 x 2 x 2 = 60

$\frac{3}{15} = \left(\frac{\_\_\_\_\_\_}{60}\right)$ $\frac{1}{20} = \left(\frac{\_\_\_\_\_\_}{60}\right)$ $\frac{2}{5} = \left(\frac{\_\_\_\_\_\_}{60}\right)$

12 3 36

35. $\frac{12}{60} + \frac{3}{60} + \frac{36}{60} = \frac{12 + 3 + 36}{60} = \left(\frac{\_\_\_\_\_\_}{60}\right)$

$\frac{51}{60}$

36. To simplify, first find the LCD:

1/6 + 2/9 + 1/12 LCD = ______

6 = 2 x 3
9 = 3 x 3
12 = 2 x 2 x 3
LCD = 2 x 2 x 3 x 3
= 36

37. $\frac{1}{6} + \frac{2}{9} + \frac{1}{12} = \left(\frac{(\_\_\_\_\_\_) + (\_\_\_\_\_\_) + (\_\_\_\_\_\_)}{36}\right)$

(Insert values but do NOT combine.)

$\frac{6 + 8 + 3}{36}$

38. $\frac{1}{6} + \frac{2}{9} + \frac{1}{12} = \frac{6 + 8 + 3}{36} = \underline{\qquad\qquad}$ (add carefully!)

$$\frac{11}{36}$$

39. $\frac{2}{15} + \frac{1}{21} + \frac{6}{35} = \frac{(\qquad) + (\qquad) + (\qquad)}{(\qquad)}$

(Insert values but do NOT combine)

$\frac{14 + 5 + 18}{105}$

40. $\frac{9}{21} + \frac{3}{16} + \frac{5}{24}$ LCD = $\underline{\qquad\qquad}$

$$\begin{array}{llllllll}
21 = & 7 \times 3 & & & & & & \\
16 = & & & & 2 \times 2 \times 2 \times 2 & & & \\
24 = & & 3 \times 2 \times 2 \times 2 & & & & & \\
\hline
\text{LCD} = & 7 \times 3 \times 2 \times 2 \times 2 \times 2 & & & & & &
\end{array}$$

41. $\frac{9}{21} + \frac{3}{16} + \frac{5}{24} = \frac{(\qquad) + (\qquad) + (\qquad)}{336}$

$\frac{144 + 63 + 70}{336}$

42. $\frac{144 + 63 + 70}{336} =$ __________

- -

$$\frac{277}{336}$$

---

43 $\frac{3}{7} + \frac{1}{4} + \frac{3}{24} =$ __________

- -

$\frac{72 + 42 + 21}{168} = \frac{135}{168} = \frac{45}{56}$

You have now completed the addition of simple fractions and should take the post-test to determine your mastery of this topic.

STOP

Tape on
Addition of Fractions

This self-study module will tell you about the Addition of Fractions, beginning with a discussion of the Least Common Denominator.

Frame number 1 gives the expression, 3/7. It is composed of two parts, a numerator and a denominator. The numerator occupies the top position, and is 3. The denominator occupies the bottom position, and it is 7.

Frame number 2 gives three fractional expressions. Identify the numerator and the denominator, and write your responses in the appropriate blanks.

The answers should be: The numerators are 3, 4, and 5; and the denominators are 8, 13, and 9.

Frame number 3 gives the fractional expression, 3/5 + 3/25. The smallest number which divides each denominator is 5.

Frame number 4 gives the fractional expression, 1/2 + 1/4. What is the smallest number which divides each denominator?

The answer is 2.

Frame 5 gives the fractional expression, 3/13 + 7/26 + 11/39. What is the smallest number that the denominators will divide?

Your answer should be 78.

The smallest number that is divisible by 8, 16, and 32, is 32. Thus, the Least Common Multiple, also called the LCM, of 8, 16, and 32, is 32. Consider the numbers 6, 16, and 24, as listed in frame 6. The Least Common Multiple is 48.

In frame 7, what is the LCM (Least Common Multiple) of 3, 4, and 5?

The answer is 60.

In the next frame, what is the LCM of 13, 26, and 39?

The answer is 78.

Some whole numbers may be written as the product of smaller whole numbers. For example, look at frame 9. Notice that in frame 9, 16 can be written in several ways. Expression c) shows 16 written as a product involving 2 x 2 x 2 x 2. This is what is called the complete factored form.

In frame 10, write in the missing numbers.

The answers are: for a) 2; for b) 6 or 2 x 3; for c) 8 or 2 x 2 x 2; for d) 4 or 2 x 2.

Addition of Fractions

A number is factored completely when it is written as a product of prime numbers. A prime number is any whole number other than one, that is divisible by itself and one.

In frame 11, there are the numbers 3, 7, 8, 5, and 12. It is seen that 3, 7, and 5 are prime numbers and that 8 and 12 can be written as products with more than one and themselves.

In frame 12, identify the prime numbers.

The answers are: a, b, and c are prime numbers; d is not a prime number.

In frame 13, complete the factorization of 24.

Your answer should be 2.

In frame 14, complete the factorization of 18.

The answer is 3 x 3 x 2.

In frame 15, factor the number 36.

The answer is 3 x 3 x 2 x 2 or any other order of those numbers such as 3 x 2 x 3 x 2.

In frame number 16, factor the number 72.

Your answer should read 3 x 3 x 2 x 2 x 2.

In frame number 17, factor the numbers 35, 16, 60, and 63.

Here are the answers: a) 35 = 5 x 7; b) 16 = 2 x 2 x 2 x 2;
c) 60 = 2 x 2 x 3 x 5; d) 63 = 3 x 3 x 7.

Complete factoring, for example the factoring of 16 which would equal 2 x 2 x 2 x 2, may be used to find the LCM (Least Common Multiple) of any series of numbers. The following example in frame 18 shows the numbers 10 and 15 completely factored.

The LCM (Least Common Multiple) of numbers 10 and 15 must contain all the prime factors without duplication of common factors. For example, the LCMs of 10 and 15 contain factors of 2, 3, and 5. Notice that 5 appears only once. As it was said before, the LCM contains all the prime factors without duplication.

In frame 19, the numbers 26 and 39 are completely factored. The LCM contains what factors?

The answer is 2, 3, and 13.

In frame 20, factor the numbers 18 and 30 completely.

Here are the correct answers: 18 = 2 x 3 x 3; 30 = 2 x 3 x 5.

The LCM contains each factor the maximum number of times it is used in either 18 or 30. The factors of 18 include how many 3s?

The answer is 2.

In the next question, the LCM must contain how many 3s?

Again, the answer is 2.

The LCM in frame 22 contains two 3s, since there are two 3s in 18. It need contain only one 2 since this factor appears only once in 18, and once in 30. Thus, the LCM must contain two 3s and one 2. What other factor of 18 and 30 must be included in the LCM? See frame 22.

5 is the answer.

In frame 23, the LCM of 18 and 30 is found. First factor 18 and 30 completely, then identify the factors and multiply to find the LCM.

Here is the answer: The LCM equals 2 x 3 x 3 x 5, which also equals 90.

In frame 24, find the LCM of 66 and 55. First factor 66 and 55 completely, then identify the factors and multiply to find the LCM.

The answer is: 66 = 2 x 3 x 11; 55 = 5 x 11. The LCM = 2 x 3 x 5 x 11 = 330.

For frame 25, find the LCM of 62 and 93.

Now, for the answers: 62 = 2 x 31; 93 = 3 x 31. The LCM = 2 x 3 x 31 = 186.

In frame 26, find the LCM of the numbers 28 and 21.

The answers are: 28 = 2 x 2 x 7; 21 = 3 x 7. The LCM = 2 x 2 x 3 x 7 = 84.

Now we will discuss the Least Common Denominator. The Least Common Denominator of a series of fractions is simply the LCM(Least Common Multiple) of the denominators of the fractions. To find the Least Common Denominator, also known as the LCD, of the fractions 4/27 and 6/45, first determine the Least Common Multiples of 27 and 45. In the next frame, the LCM of 27 and 45 is what number?

Here is the answer: 27 = 3 x 3 x 3; 45 = 3 x 3 x 5. The LCM = 3 x 3 x 3 x 5 = 135.

In frame 28, what is the Least Common Denominator or LCD of the numbers 27 and 45?

The answer is 135. Note that this answer is the same as the LCM in the preceeding question. Thus, to review: The Least Common Denominator of a series of fractions is simply the Least Common Multiple of the denominators of the fractions.

Addition of Fractions

In frame number 29, find the Least Common Denominator of 3/35, 4/55, and 1/77.

Here are the answers: 35 = 5 x 7; 55 = 5 x 11; 77 = 7 x 11. The Least Common Denominator equals 5 x 7 x 11 which, when multiplied out, equals 385.

Now, in frame 30, find the LCD of 87/52, 19/39, and 25/78.

The correct answer is: The Least Common Denominator equals 2 x 2 x 3 x 13, which equals 156.

In frame 31, find the Least Common Denominators of 5/24 + 3/16 + 8/12.

The answer is 96.

In the addition of objects, one must add the same kinds of objects. For example, 11 tequilas + 10 tequilas is equal to 21 tequilas ... which is also equal to a hangover. And if one tried to add 10 tequilas with 11 bourbons, it only equals a bigger hangover. It is readily seen that we must add only the same kinds of objects.

In frame 32, we add fractions of the same kinds, 1/5 + 3/5. The answer is equal to 4/5. Note specifically that the fractions have the same denominators. In frame 33, we add the fractions 1/7 + 2/7 + 3/7.

Note again that all three fractions, 1/7 and 2/7 and 3/7, have the same denominators. If we have fractions with different denominators, we must then find the Least Common Denominator of the fractions. See frame 34.

To answer frame 34, we must find the equivalent fractions of 3/13 in terms of the denominator 39. Proceed to frame 35.

In frame 35, we have a number of equivalencies: 1/2 is equivalent to 3/6; 7/3 is equivalent to 21/9; and 3/13 is equivalent to 9/39. It's seen from these that the values of 1/2, 7/3, and 3/13 are not changed in frame 35. The only things that are changed are the denominator and numerator.

Frame 36 demonstrates why the value has not changed. Study frame 36 carefully.

It is seen in frame 36, that in order to change the form of the fraction, we have multiplied only by the number 1, which in any case cannot change the value of the fraction.

In frame 37, find the Least Common Denominator of the three fractions, 2/7, 1/7, and 3/14.

The correct answer is 14.

Now, in frame 38, find the sum of three fractions: 4/14 + 2/14 + 3/14.

The correct answer is 9/14.

Addition of Fractions

It must be pointed out here that frames 37 and 38 represent what is known as expressions of equivalent fractions. In frame 39, find the sum of the three fractions: 1/3 + 1/13 + 1/39.

The correct answer is 17/39.

In frame number 40, you are to find the sum of 3/7 + 1/14 + 3/28.

The answer to this problem is 17/28.

You have now learned the most efficient way of adding fractions. You should now take the post-test on adding fractions to determine your competency in this area.

Name ____________________

Workbook for Cassette Tape
on
Addition of Fractions

1. $\frac{3}{7}$

2. $\frac{3}{8}$, $\frac{4}{13}$ and $\frac{5}{9}$

Numerators __________

Denominators __________

3. $\frac{3}{5} + \frac{3}{25}$

4. $\frac{1}{2} + \frac{1}{4}$

Answer __________

5. $\frac{3}{13} + \frac{7}{26} + \frac{11}{39}$

Answer __________

6. 6, 16, and 24

7. 3, 4, and 5

Answer __________

8. 13, 26, and 39

Answer __________

9.
a) 16 = 8 x 2
b) 16 = 4 x 2 x 2
c) 16 = 2 x 2 x 2 x 2

10.
a) 24 = 12 x __________
b) 24 = 4 x __________
c) 24 = 3 x __________
d) 24 = 3 x 2 x __________

---

11. a) 3 = 3 x 1
b) 7 = 7 x 1
c) 8 = 2 x 2 x 2 x 1
d) 5 = 5 x 1
e) 12 = 3 x 2 x 2 x 1

12.
a) 11 __________
b) 13 __________
c) 29 __________
d) 12 __________

13.
24 = 3 x 2 x 2 x __________

14.
18 = 3 x __________ x __________

15.
36 = __________ x __________ x __________ x __________

16.

72 = ________

17.

a) 35 = ________

b) 16 = ________

c) 60 = ________

d) 63 = ________

18.

10 = 2 x 5

15 = 3 x 5

19.

26 = 2 x 13 Answer ________

39 = 3 x 13

20.

18 = ________

30 = ________

21.

18 includes ________ 3's

LCM includes ________ 3's

22.

18 = 2 x 3 x 3 Answer ________

30 = 2 x 3 x 5

23.

18 = 2 x 3 x 3

30 = 2 x 3 x 5

LCM = ________ x ________ x ________ x ________ = ______

24.
66 = ________ x ________ x ________
55 = ________ x ________
LCM = ________ x ________ x ________ x ________

25.
62 = ________
93 = ________
LCM = ________

26.
28 = ________
21 = ________
LCM = ________

27.
27 = ________
45 = ________
LCM = ________

28.
LCD of 27 and 45 = ________

29.
35 = ________
55 = ________
77 = ________
LCD = ________

30.
Find LCD for $\frac{87}{52} + \frac{19}{39} + \frac{25}{78}$ LCD = ________

31.

$\frac{5}{24} + \frac{3}{16} + \frac{8}{12}$ LCD = ________

32.

$\frac{1}{5} + \frac{3}{5} = \frac{1 + 3}{5} = \frac{4}{5}$

33.

$\frac{1}{7} + \frac{2}{7} + \frac{3}{7} = \frac{1 + 2 + 3}{7} = \frac{6}{7}$

34.

$\frac{3}{13} + \frac{2}{39}$

$13 = 13 \times 1$

$\underline{39 = 13 \times 3 \times 1}$

LCD $= 13 \times 3 \times 1 = 39$

35.

a) $\frac{1}{2}$ is equivalent to $\frac{3}{6}$

b) $\frac{7}{3}$ is equivalent to $\frac{21}{9}$

c) $\frac{3}{13}$ is equivalent to $\frac{9}{39}$

36.

a) $\frac{1}{2} \times 1 = \frac{1}{2}$

$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$

$\frac{1}{2} = \frac{3}{6}$

b) $\frac{7}{3} \times 1 = \frac{7}{3}$

$\frac{7}{3} \times \frac{3}{3} = \frac{21}{3}$

$\frac{7}{3} = \frac{21}{3}$

c) $\frac{3}{13} \times 1 = \frac{3}{13}$

$\frac{3}{13} \times \frac{3}{3} = \frac{9}{39}$

$\frac{3}{13} = \frac{9}{39}$

37.

$\frac{2}{7} + \frac{1}{7} + \frac{3}{14}$ LCD = ________

38. $\frac{4}{14} + \frac{2}{14} + \frac{3}{14} = \frac{\quad}{14}$

39. $\frac{1}{3} + \frac{1}{13} + \frac{1}{39} = \frac{\quad}{39}$

40. $\frac{3}{7} + \frac{1}{14} + \frac{3}{28} = \underline{\qquad}$

Subtraction of Fractions

GENERAL INSTRUCTIONS

1. Before beginning this module, take the pre-test assessment to determine your knowledge for Subtraction of Fractions and their prerequisites.
2. Select one of the two instructional modes -- cassette tape or programmed instruction -- and complete the study module.
3. Take the post-test to determine your mastery of Subtraction of Fractions.

PREREQUISITES

The following prerequisites should have been mastered before undertaking this module.

1. Least Common Multiple
2. Factoring a Number into Prime Factors
3. Least Common Denominator

PRE AND POST-TEST PERFORMANCE SPECIFICATIONS

Given two different fractions you will add and record your answer on the assessment instrument. There will be five examples on the instrument. The acceptable performance level will be 80% correct responses.

INSTRUCTIONAL MODES

Choose one of two different instructional modes. Instructional mode number 1 consists of a cassette tape on how to subtract a series of fractions. Instructional mode number 2 consists of a series of programmed instructional frames on how to subtract a set of fractions.

PROGRAMMED INSTRUCTION
for
SUBTRACTION OF FRACTIONS

Before beginning this module, it is necessary to take the pre-test on Subtraction of Fractions. Programmed instruction is used as a workbook. Each frame contains one or more blanks in which you should write the numbers you feel best complete the statement in the frame.

3 x 3 = ________

9

It is necessary to use some device (a 5" x 8" index card is good) to cover the answer which appears directly below the frame that you are working. It must be pointed out that for maximum benefit, you must not look at the answer before you record your responses in the frame.

Good Luck!

SUBTRACTION OF FRACTIONS

1. Programmed texts are used, as workbooks. Each frame contains one or more blanks in which you should write the numbers or symbols you feel best complete the statement in the frame.

2 x 2 = ________

4

---

2. As soon as you have written your answer, but **not** before, look below and find the correct answer.

2 + 5 - 1 = ________

6

---

3. Compare your answer with the correct answer. If your answer is the same, proceed to the next frame; if your answer is different, study the frame and the answer until you identify your error. Programs develop progressively and you **must** understand each frame **before** you proceed to the next.

4 x 2 x 1 = ________

8

---

4. In order for you to derive the maximum benefit from this program, it is essential that you **write** your answers and check them immediately.

$$\frac{6 + 4}{2} = \_\_\_\_\_\_\_$$

5

---

5. Read all instructions carefully and work as rapidly and as accurately as you can. Remember to **write** your answers and to check them immediately.

$$(7 + 3)(2) = (\_\_\_\_\_\_\_)(2) = \_\_\_\_\_\_\_$$

$$(10)(2) = 20$$

---

6. The programmed review of subtraction of fractions begins with the next frame. The first Segment of this program consists of 16 frames.

Good Luck!

---

7. Two or more numbers multiplied together are termed **factors**, and the result of the multiplication is termed their **product**.

$2 \times 4 \times 5 = 40$ Product = _______ Factors = _______

Product = 40

Factors = 2, 4, 5

---

8. 2 x 4 x 5 = 40   Product: 40   Factors: 2, 4, 5

If the factors are divisible only by themselves and by 1, they are called **prime factors**.

Are the factors of 40 listed above all prime factors?

No; 4 is divisible by 2 as well as 4 and 1.

9. 3 x 7 x 5 = 105

Are 3, 7, and 5 the prime factors of 105? ________

Yes

10. The prime factors of 26 are 13 and 2, i.e., 26 = 13 x 2.

Similarly:
12 = 2 x 2 x ________
16 = 2 x 2 x 2 x ________
8 = ________ x ________

12 = 2 x 2 x 3
16 = 2 x 2 x 2 x 2
8 = 2 x 2 x 2

11. Recall that the Least Common Multiple (LCM) of a series of numbers is the (smallest/largest) whole number that is divisible by every member of the series.

Smallest

12. 8 = 2 x 2 x 2

12 = 2 x 2 x 3

16 = 2 x 2 x 2 x 2

LCM of 8, 12, 15 = 2 x 2 x 2x 2x __________

3

---

13. 14 = 2 x 7

21 = 3 x 7 LCM = __________ x __________ x __________ x _____

49 = 7 x 7 (Give prime factors)

---

2 x 3 x 7 x 7

---

14. The Least Common Denominator is simply the LCM of the denominators of a series of fractions.

The LCD for $\frac{1}{6}$, $\frac{3}{8}$, $\frac{5}{12}$ is __________.

---

6 = 2 x 3

8 = 2 x 2 x 2

12 = 2 x 2 x 3

LCD = 2 x 2 x 2 x 3

---

15. Find the Least Common Denominator (LCD) for the fractions in this expression:

$\frac{3}{10} + \frac{2}{5} + \frac{7}{9}$ LCD __________

$$\begin{array}{rlllll}
10 = & & 2 & & & \times 5 \\
5 = & 1 & & & & \times 5 \\
9 = & & & 3 \times 3 & & \\
\hline
\text{LCD} = & \multicolumn{5}{l}{1 \times 2 \times 3 \times 3 \times 5 = 90}
\end{array}$$

16. Find the Least Common Denominator (LCD) of $\frac{9}{24} - \frac{5}{16} - \frac{2}{21}$.

LCD = __________

$$\begin{array}{rllllll}
24 = & 2 \times 2 \times 2 & & \times 3 & \\
16 = & 2 \times 2 \times 2 \times 2 & & & \\
21 = & & & 3 \times 7 & \\
\hline
\text{LCD} = & \multicolumn{4}{l}{2 \times 2 \times 2 \times 2 \times 3 \times 7} \\
= & \multicolumn{4}{l}{336}
\end{array}$$

17. $\frac{5}{9} - \frac{1}{9} - \frac{3}{9} = \frac{(\qquad) - (\qquad) - (\qquad)}{9} = \underline{\qquad\qquad}$

$$\frac{(5) - (1) - (3)}{9} = \frac{1}{9}$$

18. $\frac{9}{10} - \frac{5}{10} - \frac{3}{10} = \frac{(\qquad) - (\qquad) - (\qquad)}{9} = \underline{\qquad\qquad}$

$$\frac{9 - 5 - 2}{10} = \frac{5}{10}$$

19. In the last two frames, you subtracted fractions that had the same denominators. The numerators of fractions may be subtracted directly only when they share the **same** denominators. When the denominators of a series of fractions are **different**, each fraction in the series must first be changed to an **equivalent fraction** with the **same** common denominator.

---

20. To express 3/8 as a fraction whose denominator is 32:

$\frac{3}{8} = \frac{3}{8} \times \frac{(\quad)}{(\quad)} = \frac{(\quad)}{32}$

- -

$\frac{3}{8} \times \frac{4}{4} = \frac{12}{32}$

---

21. $\frac{5}{14} = \frac{(\quad)}{42}$

- -

$\frac{5}{14} \times \frac{3}{3} = \frac{15}{42}$

---

22. To solve $\frac{11}{35} = \frac{?}{210}$ efficiently, consider the prime factors of the denominators: 35 = 5 x 7 210 = ________

- -

5 x 7 x 6

---

23. $35 = 5 \times 7$ $\frac{11}{35} = \frac{?}{210}$

$210 = 5 \times 6 \times 7$

__________ is the only factor 210 that is not a factor of 35. Thus, 11 and 35 must both be multiplied by __________ to obtain the equivalent fraction __________.

6

6

$\frac{66}{210}$

---

24. Since $14 = 2 \times 7$ and $42 = 2 \times 3 \times 7$,

$$\frac{3}{14} \times \frac{(\quad)}{(\quad)} = \frac{(\quad)}{42}$$

$$\frac{3}{14} \times \frac{3}{3} = \frac{9}{42}$$

---

25. To solve $\frac{15}{42} = \frac{?}{168}$

42 = __________ (Give prime

168 = __________ factors.)

$42 = 2 \times 3 \times 7$

$168 = 2 \times 2 \times 2 \times 3 \times 7$

218

26. Use prime factors to solve: $\frac{9}{44} = \frac{(\qquad)}{220}$

$$\begin{array}{rcccccccc} 44 &=& 2 &\times& 2 && &\times& 11 \\ 220 &=& 2 &\times& 2 &\times& 5 &\times& 11 \end{array}$$

$$\frac{9}{44} \times \frac{5}{5} = \frac{45}{220}$$

---

27. You have mastered the conversion of a fraction to an equivalent fraction with a different denominator. This technique will now be employed to subtract fractions.

The next several frames contain a step-by-step consideration of a sequence of operations in several examples.

---

28. To perform subtractions such as $\frac{2}{14} - \frac{3}{48} - \frac{4}{70}$,

First determine the LCD by use of prime factors.

14 = ____________

49 = ____________

70 = ____________

LCD = ____________

$$\begin{array}{rcccccccc} 14 &=& 2 && &\times& 7 && \\ 49 &=& && && 7 &\times& 7 \\ \underline{70} &=& \underline{2} &\underline{\times}& \underline{5} &\underline{\times}& \underline{7} && \\ \text{LCD} &=& 2 &\times& 5 &\times& 7 &\times& 7 \\ &=& 490 &&&&&& \end{array}$$

29. $\frac{2}{14} - \frac{3}{49} - \frac{4}{70}$ $(490 = 2 \times 5 \times 7 \times 7)$

$\frac{2}{14} = \frac{70}{490}$ $\frac{3}{49} = \frac{(\qquad)}{490}$ $\frac{4}{70} = \frac{(\qquad)}{490}$

- -

$$\frac{3}{49} \times \left(\frac{2 \times 5}{2 \times 5}\right) = \frac{30}{490}$$

$$\frac{4}{70} \times \frac{7}{7} = \frac{28}{490}$$

---

30. Note that $\frac{2 \times 2 \times 3}{2 \times 3 \times 5}$ may be regrouped as $\frac{2}{2} \times \frac{3}{3} \times \frac{2}{5}$.

Since 2/2 = __________ and 3/3 = __________, the elimination of these factors common to numerator and denominator (will/will not) change the value of the fraction.

- -

1, 1

will not

---

31. $\frac{2 \times 2 \times 3}{2 \times 3 \times 5} \Longrightarrow \frac{2}{2} \times \frac{3}{3} \times \frac{2}{5} \Longrightarrow \frac{2}{5}$

$\frac{3 \times 4}{4 \times 7} \Longrightarrow \frac{(\qquad)}{(\qquad)} \times \frac{(\qquad)}{(\qquad)} \Longrightarrow$ __________

- -

$$\frac{4}{4} \times \frac{3}{7} \Longrightarrow \frac{3}{7}$$

32. The elimination of factors common to both numerator and denominator of a fraction is termed reducing.

Reduce: $\frac{3 \times 7 \times 11}{4 \times 5 \times 7 \times 11} = \frac{(\qquad)}{\;} \times \frac{(\qquad)}{\;} \times \frac{(\qquad)}{(\qquad)(\qquad)}$

$\Longrightarrow$ ________

$$\frac{7}{7} \times \frac{11}{11} \times \frac{3}{4 \times 5} \Longrightarrow \frac{3}{4 \times 5}$$

33. Recall that the factors of 168 are 2 x 2 x 2 x 3 x 7

Does 87 contain any of these same factors? Which? ________

Yes, 3 (87 = 3 x 29)

34. Reduce: $\frac{87}{168} =$ ________

$$\frac{87}{168} = \frac{\cancel{3} \times 29}{2 \times 2 \times \cancel{3} \times 7}$$

$$= \frac{29}{42}$$

35. To reduce a fraction such as 70/105: (1) express numerator and denominator as prime factors, and (2) reduce where possible.

$\frac{70}{105} = \frac{2 \times 5 \times 7}{3 \times 5 \times 7} =$ ________

$$\frac{2}{3}$$

36. $\frac{14}{15} - \frac{1}{20} - \frac{3}{5}$ LCD = 5 x 3 x 2 x 2 = 60

$\frac{14}{15} = \frac{(\qquad)}{60}$ $\frac{1}{20} = \frac{(\qquad)}{60}$ $\frac{2}{5} = \frac{(\qquad)}{60}$

56 3 36

37. $\frac{56}{60} - \frac{3}{60} - \frac{36}{60} = \frac{56 - 3 - 36}{60} = \frac{(\qquad)}{60}$

$\frac{17}{60}$

38. To simplify, first find the LCD:

$\frac{7}{6} - \frac{4}{9} - \frac{5}{12}$ LCD = __________

$$\begin{array}{rl} 6 = & 2 \qquad\quad \times 3 \\ 9 = & \qquad\qquad\; 3 \times 3 \\ 12 = & 2 \times 2 \times 3 \\ \hline \text{LCD} = & 2 \times 2 \times 3 \times 3 \\ = & 36 \end{array}$$

39. $\frac{7}{6} - \frac{4}{9} - \frac{5}{12} = \frac{(\qquad) - (\qquad) - (\qquad)}{36}$

(Insert values but do NOT combine.)

$\frac{42 - 16 - 15}{36}$

40. $\frac{7}{6} - \frac{4}{9} - \frac{5}{12} = \frac{42 - 16 - 15}{36} = \_\_\_\_\_\_$ (subtract carefully!)

$$\frac{11}{36}$$

41. $\frac{7}{15} - \frac{2}{21} - \frac{11}{35} = \frac{(\qquad) - (\qquad) - (\qquad)}{(\qquad)}$ (Insert values but do NOT combine)

$$\frac{245 - 50 - 175}{105}$$

42. $\frac{9}{21} - \frac{3}{16} - \frac{5}{24}$ LCD = ______

$$\begin{array}{rl} 21 = & 7 \times 3 \\ 16 = & \qquad\quad 2 \times 2 \times 2 \times 2 \\ 24 = & \quad 3 \times 2 \times 2 \times 2 \\ \hline \text{LCD} = & 7 \times 3 \times 2 \times 2 \times 2 \times 2 \end{array}$$

43. $\frac{9}{21} - \frac{3}{16} - \frac{5}{24} = \frac{(\qquad) - (\qquad) - (\qquad)}{336}$

$$\frac{144 - 63 - 70}{336}$$

44. $\frac{144 - 63 - 70}{336}$ = ____________

$$\frac{11}{336}$$

---

45. $\frac{6}{7} - \frac{1}{4} - \frac{3}{24}$ = ____________

$$\frac{144 - 42 - 21}{168} = \frac{81}{168} = \frac{27}{56}$$

You have now completed the subtraction of simple fractions and should take the post-test to determine your mastery of this topic

STOP

Tape on
Subtraction of Fractions

This self-study module will tell you about the Subtraction of Fractions, beginning with a discussion of the Least Common Denominator.

Frame number 1 gives the expression, 3/7. It is composed of two parts, a numerator and a denominator. The numerator occupies the top position, and is 3. The denominator occupies the bottom position, and it is 7.

Frame number 2 gives three fractional expressions. Identify the numerator and the denominator, and write your responses in the appropriate blanks.

The answers should be: The numerators are 3, 4, and 5; and the denominators are 8, 13, and 9.

Frame number 3 gives the fractional expression, 3/5 + 3/25. The smallest number which divides each denominator is 5.

Frame number 4 gives the fractional expression, 1/2 + 1/4. What is the smallest number which divides each denominator?

The answer is 2.

Frame 5 gives the fractional expression, 3/13 + 7/26 + 11/39. What is the smallest number that the denominators will divide?

Your answer should be 78.

The smallest number that is divisible by 8, 16, and 32, is 32. Thus, the Least Common Multiple, also called the LCM, of 8, 16, and 32, is 32. Consider the numbers 6, 16, and 24, as listed in frame 6. The Least Common Multiple is 48.

In frame 7, what is the LCM (Least Common Multiple) of 3, 4, and 5?

The answer is 60.

In the next frame, what is the LCM of 13, 26, and 39?

The answer is 78.

Some whole numbers may be written as the product of smaller whole numbers. For example, look at frame 9. Notice that in frame 9, 16 can be written in several ways. Expression c) shows 16 written as a product involving 2 x 2 x 2 x 2. This is what is called the complete factored form.

In frame 10, write in the missing numbers.

The answers are: for a) 2; for b) 6 or 2 x 3; for c) 8 or 2 x 2 x 2; for d) 4 or 2 x 2.

Subtraction of Fractions

A number is factored completely when it is written as a product of prime numbers. A prime number is any whole number other than one, that is divisible by itself and one.

In frame 11, there are the numbers 3, 7, 8, 5, and 12. It is seen that 3, 7, and 5 are prime numbers and that 8 and 12 can be written as products with more than one and themselves.

In frame 12, identify the prime numbers.

The answers are: a, b, and c are prime numbers; d is not a prime number.

In frame 13, complete the factorization of 24.

Your answer should be 2.

In frame 14, complete the factorization of 18.

The answer is 3 x 3 x 2.

In frame 15, factor the number 36.

The answer is 3 x 3 x 2 x 2 or any other order of those numbers such as 3 x 2 x 3 x 2.

In frame number 16, factor the number 72.

Your answer should read 3 x 3 x 2 x 2 x 2.

In frame number 17, factor the numbers 35, 16, 60, and 63.

Here are the answers: a) 35 = 5 x 7; b) 16 = 2 x 2 x 2 x 2; c) 60 = 2 x 2 x 3 x 5; d) 63 = 3 x 3 x 7.

Complete factoring, for example the factoring of 16 which would equal 2 x 2 x 2 x 2, may be used to find the LCM (Least Common Multiple) of any series of numbers. The following example in frame 18 shows the numbers 10 and 15 completely factored.

The LCM (Least Common Multiple) of numbers 10 and 15 must contain all the prime factors without duplication of common factors. For example, the LCMs of 10 and 15 contain factors of 2, 3, and 5. Notice that 5 appears only once. As it was said before, the LCM contains all the prime factors without duplication.

In frame 19, the numbers 26 and 39 are completely factored. The LCM contains what factors?

The answer is 2, 3, and 13.

In frame 20, factor the numbers 18 and 30 completely.

Here are the correct answers: 18 = 2 x 3 x 3; 30 = 2 x 3 x 5.

Subtraction of Fractions

The LCM contains each factor the maximum number of times it is used in either 18 or 30. The factors of 18 include how many 3s?

The answer is 2.

In the next question, the LCM must contain how many 3s?

Again, the answer is 2.

The LCM in frame 22 contains two 3s, since there are two 3s in 18. It need contain only one 2 since this factor appears only once in 18, and once in 30. Thus, the LCM must contain two 3s and one 2. What other factor of 18 and 30 must be included in the LCM? See frame 22.

5 is the answer.

In frame 23, the LCM of 18 and 30 is found. First factor 18 and 30 completely, then identify the factors and multiply to find the LCM.

Here is the answer: The LCM equals 2 x 3 x 3 x 5, which also equals 90.

In frame 24, find the LCM of 66 and 55. First factor 66 and 55 completely, then identify the factors and multiply to find the LCM.

The answer is: 66 = 2 x 3 x 11; 55 = 5 x 11. The LCM = 2 x 3 x 5 x 11 = 330.

For frame 25, find the LCM of 62 and 93.

Now, for the answers: 62 = 2 x 31; 93 = 3 x 31. The LCM = 2 x 3 x 31 = 186.

In frame 26, find the LCM of the numbers 28 and 21.

The answers are: 28 = 2 x 2 x 7; 21 = 3 x 7. The LCM = 2 x 2 x 3 x 7 = 84.

Now we will discuss the Least Common Denominator. The Least Common Denominator of a series of fractions is simply the LCM(Least Common Multiple) of the denominators of the fractions. To find the Least Common Denominator, also known as the LCD, of the fractions 4/27 and 6/45, first determine the Least Common Multiples of 27 and 45. In the next frame, the LCM of 27 and 45 is what number?

Here is the answer: 27 = 3 x 3 x 3; 45 = 3 x 3 x 5. The LCM = 3 x 3 x 3 x 5 = 135.

In frame 28, what is the Least Common Denominator or LCD of the numbers 27 and 45?

The answer is 135. Note that this answer is the same as the LCM in the preceeding question. Thus, to review: The Least Common Denominator of a series of fractions is simply the Least Common Multiple of the denominators of the fractions.

Subtraction of Fractions

In frame number 29, find the Least Common Denominator of 3/35, 4/55, and 1/77.

Here are the answers: 35 = 5 x 7; 55 = 5 x 11; 77 = 7 x 11. The Least Common Denominator equals 5 x 7 x 11 which, multiplied out, equals 385.

Now, in frame 30, find the LCD of 87/52, 19/39, and 25/78.

The correct answer is: The Least Common Denominator equals 2 x 2 x 3 x 13, which equals 156.

In frame 31, find the LCD of 12/13 - 2/39 - 1/26.

The answer is 78. In subtraction, one must subtract the same kinds of objects. For example, 20 tequilas - 5 tequilas is equal to 15 tequilas, which also equals to a hangover. On the other hand, if you tried to subtract 11 bourbons from 20 tequilas, they can only be confusing and probably would wind up as a hangover, too. It's readily seen then, that we must subtract only the same kinds of objects.

In frame 32, we subtract 1/5 from 3/5. The answer will be 2/5. Note in frame 32, that both fractions have the same denominator.

Now, in frame 33, let's see how you do at subtracting 2/7 and 1/7 from 6/7.

The answer is 3/7. Note again that all three fractions 6/7, 1/7, and 2/7 have the same denominator. It's been seen that in order to do subtraction with fractions, you must have the same denominators.

Now, if we have fractions with different denominators, we must find the lowest common denominator of the fractions. Try your hand at frame 34.

The lowest common denominator in frame 34, is equal to 13 x 3 x 1, which equals 39. Now we must find the equivalent fractions of 3/13 in terms of the denominator 39.

In frame 35, we see that 1/2 is equivalent to 3/6; 7/3 is equivalent to 21/9; and 3/13 is equivalent to 9/39. It's seen that the values of 1/2, 7/3 and 3/13 are not changed in frame 35, the only thing that has changed is the denominator and the numerator.

Frame 36 demonstrates why the value has not changed. In problem "a", we see that 1/2 x 1 equals 1/2 and 1/2 x 3/3, which of course is also 1, equals 3/6. So 1/2 will be equal to 3/6. In problem b, 7/3 x 1 equals 7/3. 7/3 x 3/3, of course which equals 1, equals 21/9 and 7/3 equals 21/9. In the third problem 3/13 x 1 equals 3/13. 3/13 x 3/3 equals 9/39. 3/13 equals 9/39.

We can see from frame 36, that in order to change the form of the fraction, we have multiplied only by the number 1, which in any case cannot change the value of the fraction.

Now in frame 37, find the Lowest Common Denominator of the three fractions 5/7 - 1/7 - 3/14.

The answer is, the Lowest Common Denominator is 14.

Subtraction of Fractions

In frame 38, do the subtraction of the three fractions 10/14 - 2/14 - 3/14. I might point out that frames 37 and 38 represent expressions for equivalent fractions.

The answer will be 5.

In frame 39, find the difference of the three fractions 1/3 - 1/13 - 1/39.

The answer is 9/39.

In frame 40, find the difference of the three fractions 4/7 - 1/14 - 3/28.

The answer is 11/28.

You have not completed the most efficient way of subtracting fractions. You should now take the post-test on subtracting fractions to determine just how well you have learned this concept.

Name ____________________

Workbook for Cassette Tape
on
Subtraction of Fractions

1.

$\frac{3}{7}$

2.

$\frac{3}{8}, \frac{4}{13},$ and $\frac{5}{9}$

Numerators __________

Denominators __________

3.

$\frac{3}{5} + \frac{3}{25}$

4.

$\frac{1}{2} + \frac{1}{4}$

Answer __________

5.

$\frac{3}{13} + \frac{7}{26} + \frac{11}{39}$

6.

6, 16, and 24

7.

3, 4, and 5

Answer __________

8.

13, 26 and 39

Answer __________

9.

a) 16 = 8 x 2

b) 16 = 4 x 2 x 2

c) 16 = 2 x 2 x 2 x 2

10.

a) 24 = 12 x ________

b) 24 = 4 x ________

c) 24 = 3 x ________

d) 24 = 3 x 2 x ________

11.

a) 3 = 3 x 1

b) 7 = 7 x 1

c) 8 = 2 x 2 x 2 x 1

d) 5 = 5 x 1

e) 12 = 3 x 2 x 2 x 1

12.

a) 11 ________

b) 13 ________

c) 29 ________

d) 12 ________

13.

24 = 3 x 2 x 2 x ________

14.

18 = 3 x ________ x ________

15.

36 = ________ x ________ x ________ x ________

16.

72 = ______

17.

a) 35 = ______

c) 16 = ______

d) 60 = ______

e) 63 = ______

18.

10 = 2 x 5

15 = 3 x 5

19.

26 = 2 x 13 Answer______

39 = 3 x 13

20.

18 = ______

30 = ______

21.

18 includes ______ 3's

LCM includes ______ 3's

22.

18 = 2 x 3 x 3 Answer ______

30 = 2 x 3 x 5

23.

18 = 2 x 3 x 3

30 = 2 x 3 x 5

LCM = ______ x ______ x ______ x ______ = ______

24.

66 = ________ x ________ x ________

55 = ________ x ________

LCM = ________ x ________ x ________ x ________

25.

62 = ________

93 = ________

LCM = ________

26.

28 = ________

21 = ________

LCM = ________

27.

27 = ________

45 = ________

LCM = ________

28.

LCD of 27 and 45 = ________

29.

35 = ________

55 = ________

LCD = ________

30.

Find LCD for $\frac{87}{52} + \frac{19}{39} + \frac{25}{78}$ LCD = ________

31

$\frac{12}{13} - \frac{2}{39} - \frac{1}{26}$ LCD ________

32.

$\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$

33.

$\frac{6}{7} - \frac{2}{7} - \frac{1}{7} = \frac{6 - 2 - 1}{7} = \frac{3}{7}$

34.

$\frac{3}{13} - \frac{2}{39}$

$$\begin{array}{l} 13 = 13 \times \phantom{3 \times{}} 1 \\ 39 = 13 \times 3 \times 1 \\ \hline \text{LCD} = 13 \times 3 \times 1 = 39 \end{array}$$

35.

a) $\frac{1}{2}$ is equivalent to $\frac{3}{6}$

b) $\frac{7}{3}$ is equivalent to $\frac{21}{9}$

c) $\frac{3}{13}$ is equivalent to $\frac{9}{39}$

36.

a) $\frac{1}{2} \times 1 = \frac{1}{2}$

$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$

$\frac{1}{2} = \frac{3}{6}$

b) $\frac{7}{3} \times 1 = \frac{7}{3}$

$\frac{7}{3} \times \frac{3}{3} = \frac{21}{9}$

$\frac{7}{3} = \frac{21}{9}$

c) $\frac{3}{13} \times 1 = \frac{3}{13}$

$\frac{3}{13} \times \frac{3}{3} = \frac{9}{39}$

$\frac{3}{13} = \frac{9}{39}$

37.

$\frac{5}{7} - \frac{1}{7} - \frac{3}{14}$ LCD = ________

38.

$$\frac{10}{14} - \frac{2}{14} - \frac{3}{14} = \frac{\qquad\qquad}{14}$$

39.

$$\frac{1}{3} - \frac{1}{13} - \frac{1}{39} = \frac{\qquad\qquad}{39}$$

40.

$$\frac{4}{7} - \frac{1}{14} - \frac{3}{28} = \underline{\qquad\qquad}$$

Multiplication of Fractions

GENERAL INSTRUCTIONS

1. Before beginning this module, take the pre-test assessment to determine your knowledge for Multiplication of Fractions and their prerequisites.
2. Select one of the two instructional modes -- cassette tape or programmed instruction -- and complete the study module.
3. Take the post-test to determine your mastery of Multiplication of Fractions.

PREREQUISITES

The following prerequisites should have been mastered before undertaking this module.

1. Factoring a Number into Prime Factors
2. Equivalent Fractions
3. Reduction of Fractions

PRE AND POST-TEST PERFORMANCE SPECIFICATIONS

Given two different fractions you will multiply and record your answer on the assessment instrument. There will be five examples on the instrument. The acceptable performance level will be 80% correct responses.

INSTRUCTIONAL MODES

Choose one of two different instructional modes. Instructional mode number 1 consists of a cassette tape on how to multiply a series of fractions. Instructional mode number 2 consists of a series of programmed instructional frames on how to multiply a set of fractions.

PROGRAMMED INSTRUCTION
for
MULTIPLICATION OF FRACTIONS

Before beginning this module, it is necessary to take the pre-test on Multiplication of Fractions. Programmed instruction is used as a workbook. Each frame contains one or more blanks in which you should write the numbers you feel best complete the statement in the frame.

3 x 3 = ________

9

It is necessary to use some device (a 5" x 8" index card is good) to cover the answer which appears directly below the frame that you are working. It must be pointed out that for maximum benefit, you must not look at the answer before you record your responses in the frame.

Good Luck!

MULTIPLICATION OF FRACTIONS

1. Throughout this program, questions you are asked to answer will appear in boxes like this one; these boxes are called frames. The next frame appears below.

The answers appear in this position on every page.

2. Programmed texts are used, not read. Each frame contains one or more blanks in which you should write the numbers or symbols you feel best complete the statement in the frame.

2 x 2 = ________

4

3. As soon as you have written your answer, but not before, look below at the answer.

2 + 5 - 1 = ________

6

4. Compare your answer with the correct answer. If [illegible] answer is the same, proceed to the next frame; if your answer is different, study the frame and the answer until you identify your error. Programs develop progressively and you must understand each frame before you proceed to the next.

4 x 2 x 1 = ________

8

5. In order for you to derive the maximum benefit from this program, it is essential that you write your answers and check them immediately.

$\frac{4 + 5}{3} =$ ________

3

---

6. Read all instructions carefully and work as rapidly and as accurately as you can. Remember to write your answers and to check them immediately!

(9 - 4) (3) = (________) (3) = ________

---

(5)(3) = 15

---

7. The program in this Instruction reviews a number of techniques that facilitate the rapid and efficient multiplication of fractions and fractional expressions.

Good Luck!

---

8. To multiply two fractions $\frac{a}{b} \times \frac{c}{d}$, simply multiply the numerators together and multiply the denominators together.

$\frac{a}{b} \times \frac{c}{d} = \frac{(\qquad)}{(\qquad)}$

$\frac{a \times c}{b \times d}$ or $\frac{ac}{bd}$

---

9. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$

$\frac{3}{7} \times \frac{5}{8} = \frac{3 \times 5}{7 \times 8} = $ __________

$$\frac{15}{56}$$

10. $\frac{6}{11} \times \frac{7}{5} = \frac{(\qquad)}{(\qquad)} \times \frac{(\qquad)}{(\qquad)} = \frac{(\qquad)}{(\qquad)}$

$$\frac{6 \times 7}{11 \times 5} = \frac{42}{55}$$

11. This principle may be applied to the multiplication of any series of fractions: $\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} = \frac{ace}{bdf}$

$\frac{2}{5} \times \frac{3}{7} \times \frac{6}{11} = $ __________

$$\frac{2 \times 3 \times 6}{5 \times 7 \times 11} = \frac{36}{385}$$

12. The simple steps of changing each term to prime factors and then reducing will eliminate a great deal of tedious multiplication and division that often causes careless errors. You will benefit considerably, therefore, by mastering the approach developed in the following segment. This approach, foolproof when properly applied, is based on the use of prime factors.

13. A **prime number** is any whole number (other than 1) that is divisible only by itself and by 1.

Which of the following numbers is/are prime?

3 6 7 13 15 18 __________

Only 3, 7, and 15

14. Two or more numbers multiplied together are termed **factors**, and the result of the multiplication is termed their **product**.

2 x 6 x 5 = 60

Factors = __________

Product = __________

2, 6, 5
60

15. A number is reduced to its **prime factors** when it is represented as the product of a series of **prime** numbers; e.g.,

26 = 2 x 13   18 = 2 x 3 x 3

27 = __________ x __________ x __________

2, 3, 3

16. Rewrite as prime factors:

16 = 2 x 2 x __________ x __________   42 = __________ x __________ x __________

30 = 2 x __________ x __________   17 = __________ x __________

16 = 2 x 2 x 2 x 2

30 = 2 x 3 x 5

42 = 2 x 3 x 7

7 is already prime

17. To multiply fractions, first rewrite each number as its prime factors.

For Example: $\frac{4}{6} \times \frac{9}{16} \Longrightarrow \frac{2 \times 2}{3 \times 2} \times \frac{3 \times 3}{2 \times 2 \times 2 \times 2}$

Give prime factors for: $\frac{12}{30} \times \frac{10}{15} \Longrightarrow \frac{(\qquad)}{(\qquad)} \times \frac{(\qquad)}{(\qquad)}$

$$\frac{2 \times 2 \times 3}{2 \times 3 \times 5} \times \frac{2 \times 5}{3 \times 5}$$

18. Note that $\frac{2 \times 2 \times 3}{2 \times 3 \times 5}$ may be regrouped as $\frac{2}{2} \times \frac{3}{3} \times \frac{2}{5}$.

Since 2/2 = __________ and 3/3 = __________, the elimination of factors common to both numerator and denominator (will/will not) change the value of the fraction.

1,1

will not

19. $\frac{2 \times 2 \times 3}{2 \times 3 \times 5} \Longrightarrow \frac{2}{2} \times \frac{3}{3} \times \frac{2}{5} \Longrightarrow \frac{2}{5}$

$\frac{2 \times 5}{3 \times 5} \Longrightarrow \frac{(\qquad)}{(\qquad)} \times \frac{(\qquad)}{(\qquad)} \Longrightarrow$ __________

$$\frac{5}{5} \times \frac{2}{3} \Longrightarrow \frac{2}{3}$$

20. The elimination of factors common to both numerator and denominator of a fraction is termed **reducing**.

Reducing: $\frac{3 \times 7 \times 13}{3 \times 5 \times 7 \times 11} \Longrightarrow \frac{(\quad)}{(\quad)} \times \frac{(\quad)}{(\quad)} \times \frac{(\quad)}{(5)(\quad)}$

$\Longrightarrow$ ________

$$\frac{3}{3} \times \frac{7}{7} \times \frac{13}{5 \times 11}$$

$$\Longrightarrow \frac{13}{5 \times 11}$$

---

21. It is not actually necessary to regroup the prime factors of numerator and denominator when reducing a fraction.

Reduce directly: $\frac{2 \times 2 \times 7 \times 13}{7 \times 13 \times 13} \Longrightarrow$

Do NOT Multiply.
Keep prime factors.

$$\frac{2 \times 2 \times \cancel{7} \times \cancel{13}}{\cancel{7} \times \cancel{13} \times 13} \Longrightarrow \frac{2 \times 2}{13}$$

---

22. The second step in multiplying fractions is to reduce each fraction, if possible.

$\frac{12}{30} \times \frac{10}{35} \Longrightarrow \frac{2 \times 2 \times 3}{2 \times 3 \times 5} \times \frac{2 \times 5}{5 \times 7} \Longrightarrow \frac{\cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 5} \times \frac{2 \times \cancel{5}}{\cancel{5} \times 7}$

$\Longrightarrow \frac{(\quad)}{(\quad)} \times \frac{(\quad)}{(\quad)}$

$$\frac{2}{5} \times \frac{2}{7}$$

23. $\frac{12}{30} \times \frac{10}{35} \Longrightarrow \frac{\cancel{2} \times 2 \times \cancel{3}}{\cancel{2} \times \cancel{3} \times 5} \times \frac{2 \times \cancel{5}}{\cancel{5} \times 7} \Longrightarrow \frac{2}{5} \times \frac{2}{7}$

$\Longrightarrow \frac{(\qquad\qquad)(\qquad\qquad)}{(\qquad\qquad)(\qquad\qquad)} = \underline{\qquad\qquad}$

- -

$$\frac{2 \times 2}{5 \times 7} = \frac{4}{35}$$

---

24. Try another example step by step.

$\frac{21}{28} \times \frac{12}{20} \Longrightarrow \underline{\qquad\qquad} \times \underline{\qquad\qquad}$

Give prime factors. Do not reduce.

- -

$$\frac{3 \times 7}{2 \times 2 \times 7} \times \frac{2 \times 2 \times 3}{5 \times 2 \times 2}$$

---

25. $\frac{21}{20} \times \frac{12}{20} \Longrightarrow \frac{3 \times \cancel{7}}{2 \times 2 \times \cancel{7}} \times \frac{\cancel{2} \times \cancel{2} \times 3}{\cancel{2} \times \cancel{2} \times 5}$ (each fraction reduced)

$\Longrightarrow \frac{(\qquad\qquad)}{(\qquad\qquad)}$

(Rewrite remaining prime factors in a single fraction.)

- -

$$\frac{3 \times 3}{2 \times 2 \times 5}$$

---

26. $\frac{21}{28} \times \frac{12}{20} \Longrightarrow \frac{3 \times \cancel{7}}{2 \times 2 \times \cancel{7}} \times \frac{\cancel{2} \times \cancel{2} \times 3}{\cancel{2} \times \cancel{2} \times 5}$

$\Longrightarrow \frac{3 \times 3}{2 \times 2 \times 5} \Longrightarrow$ ________ (New fraction reduced.)

$$\frac{9}{20}$$

27. $\frac{28}{30} \times \frac{15}{33} \Longrightarrow$ ________ x ________

Give prime factors only.
Do not reduce.

$$\frac{2 \times 2 \times 7}{2 \times 3 \times 5} \times \frac{3 \times 5}{3 \times 11}$$

28. $\frac{28}{30} \times \frac{15}{33} \Longrightarrow \frac{\cancel{2} \times 2 \times 7}{\cancel{2} \times 3 \times 5} \times \frac{\cancel{3} \times 5}{\cancel{3} \times 11} \Longrightarrow \frac{(\qquad)}{(\qquad)}$ (Give prime factors.)

Reduce again to obtain ________.
(prime factors)

$$\frac{3 \times 7 \times 5}{3 \times 5 \times 11}$$

$$= \frac{2 \times 7}{3 \times 11} = \frac{14}{33}$$

29. $\frac{35}{42} \times \frac{34}{55} \Longrightarrow \frac{5 \times \cancel{7}}{2 \times 3 \times \cancel{7}} \times \frac{2 \times 17}{5 \times 11} \Longrightarrow \frac{\cancel{5} \times \cancel{2} \times 17}{\cancel{2} \times 3 \times \cancel{5} \times 11}$

$$\Longrightarrow \frac{17}{3 \times 11} = \frac{17}{33}$$

Give prime factors and reduce each fraction separately.

$\frac{12}{15} \times \frac{10}{14} = \frac{(\qquad)}{(\qquad)} \times \frac{(\qquad)}{(\qquad)}$

$$\frac{2 \times 2 \times \cancel{3}}{\cancel{3} \times 5} \times \frac{\cancel{2} \times 5}{\cancel{2} \times 7}$$

30. $\frac{24}{28} \times \frac{10}{14} \Longrightarrow \frac{\cancel{2} \times \cancel{2} \times 2 \times 3}{\cancel{2} \times \cancel{2} \times 7} \times \frac{\cancel{2} \times 5}{\cancel{2} \times 7} \Longrightarrow \frac{(\qquad)}{(\qquad)}$ (Give prime factors.)

Reduce again to obtain $\frac{(\qquad)}{(\qquad)} =$ ________.

(prime factors)

$$\frac{2 \times 3 \times \cancel{5}}{\cancel{5} \times 7}$$

$$\Longrightarrow \frac{2 \times 3}{7} = \frac{6}{7}$$

31. Give prime factors for each fraction and reduce. Do not multiply.

$\frac{8}{27} \times \frac{70}{16} \times \frac{18}{35} = \frac{(\qquad)}{(\qquad)} \times \frac{(\qquad)}{(\qquad)} \times \frac{(\qquad)}{(\qquad)}$

$$\left(\frac{2 \times 2 \times 2}{3 \times 3 \times 3}\right) \times \left(\frac{\cancel{2} \times 5 \times 7}{\cancel{2} \times 2 \times 2 \times 2}\right) \times \left(\frac{2 \times 3 \times 3}{5 \times 7}\right)$$

32. $\frac{8}{27} \times \frac{70}{16} \times \frac{18}{35} = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} \times \frac{5 \times 7}{2 \times 2 \times 2}$ [illegible] $\frac{2 \times 3 \times \text{[illegible]}}{5 \times \text{[illegible]}}$

$= \frac{(\qquad)}{(\qquad)}$

= ____________

$$\frac{2 \times \cancel{2} \times \cancel{2} \times \cancel{5} \times \cancel{7} \times \cancel{2} \times \cancel{3} \times \cancel{3}}{3 \times \cancel{3} \times \cancel{3} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{5} \times \cancel{7}}$$

$$\frac{2}{3}$$

---

33. Use the technique of prime factoring to multiply the following. Proceed carefully and step by step.

$\frac{12}{18} \times \frac{27}{45} =$ ____________

$$\frac{\cancel{2} \times 2 \times \cancel{3}}{2 \times \cancel{3} \times 3} \times \frac{\cancel{3} \times \cancel{3} \times 3}{\cancel{3} \times \cancel{3} \times 5}$$

$$= \frac{2 \times \cancel{3}}{\cancel{3} \times 5} = \frac{2}{5}$$

You have completed the unit on the multiplication of simple fractions and you should now take the post-test to check your mastery of this topic.

STOP

Tape on
Multiplication of Fractions

This self-instructional tape will deal with the multiplication of fractions.

To multiply two fractions, you simply multiply the numerators of the fractions together and multiply the denominators together.

In frame 1, 1/5 is multiplied by 2/3, and the result is 2/15.

In frame number 2, multiply the following fractions: a) 1/7 x 1/3 = _____; b) 3/11 x 3/5 = _____; c) 11/12 x 5/9 = _____.

Here are the answers: The answer to a) is 1/21; The answer to b) is 9/55; The answer to c) is 55/108.

In frame number 3, multiply the following fractions: a) 3/8 x 5/7 x 1/2; b) 5/12 x 7/11 x 1/4; and c) 1/4 x 3/7 x 5/13.

Here are the answers: The answer to a) is 15/112; The answer to b) is 35/528; The answer to c) is 15/364.

You have now seen the principle of the multiplication of fractions at work. To multiply fractions, we simply multiply the numerators together and multiply the denominators together.

This principle may be applied to the multiplication of any series of fractions. For example, look at frame 4: a/b x c/d x e/f x g/h = (a x c x e x g)/(b x d x f x h).

The simple steps of changing each term in the numerator and the denominator to prime factors and then reducing will eliminate a great deal of tedious multiplication that often causes errors. You will benefit considerably, therefore, by mastering the approach developed in the following frames.

A prime number is any whole number, other than one, that is divisible only by itself and one.

In frame 5, determine whether the numbers 3, 6, 7, and 12 are prime numbers or not prime numbers.

Here are the answers: a) is a prime number; b) 6 is not a prime number because you can multiply 1 x 2 x 3; c) is a prime number; and 12 is not a prime number because you can multiply 1 x 2 x 2 x 3.

It's seen in that frame that the numbers 6 and 12 can be divided by 1, 2, and 3 and that consequently they are not prime numbers. Two or more numbers multiplied together are termed factors and the result of their multiplication is termed their product.

In frame number 6, we have the following multiplication: 4 x 3 x 6 = 72. The factors are 4, 3, 6 and their product is 72.

Multiplication of Fractions

A number is reduced to its prime factors when it is represented as the product of a series of prime numbers. For example, in frame 7: 26 = 2 x 13; 27 = 3 x 3 x 3. The product of 26 is represented by the multiplication of 2 and 13. These are both prime factors. The product of 27 is represented by 3 x 3 x 3, which are all prime factors.

In frame number 8, identify which of the numbers are prime numbers.

Here are the answers: 11 is a prime number, because 1 x 11 = 11; 15 is not a prime number, because 1 x 3 x 5 = 15; 17 is a prime number, because 1 x 17 =17; 23 is also a prime number because 1 x 23 = 23.

In frame number 9, identify the products and factors of 27 and 105.

Here are the answers: The factors are 1, 2, 11, 3, 5, and 7. The products are 22 and 105.

In frame number 10, rewrite the numbers as prime factors.

Here are the answers: The answer to problem a) 12 = 2 x 2 x 3; problem b) 30 = 2 x 3 x 5; problem c) 25 = 5 x 5; problem d) 7 is already a prime number.

To multiply fractions, first rewrite each number as its prime factors. For example, look at frame number 11: 4/6 x 9/16 = {(2 x 2)/(3 x 2)} x {(3 x 3)/(2 x 2 x 2 x 2)}.

In frame 12, give the factors for 12/30 x 10/35.

Here is the answer: {(2 x 2 x 3)/(2 x 3 x 5)} x {(2 x 5)/(5 x 7)}.

Note that the answer given in 12 is also given in frame 13. And that (2 x 2 x 3)/(2 x 3 x 5) may be regrouped as 2/2 x 3/3 x 2/5. Looking at frame 13 again, since 2/2 equals 1, and 3/3 equals 1,what we have is 1 x 1 x 2/5. Therefore, the result in frame 13 is 2/5. This is clarified for you in frame 14.

The elimination of factors common to numerators and denominators does not change the value of a fraction. The only change is its form. For instance, 2/2 equals 1, 3/3 equals 1, and if you multiply any number by 1, you have a result that is equal to that number. As an example, look at frame 15. Problem a) 1 x 1 x 5 = 5. In problem b) 2/2 x 3/3 x 5/1 = 1 x 1 x 5 = 5. As it is seen, the value is not changed. The elimination of factors common to both the numerator and denominator of a fraction is termed reducing.

In frame 16, reduce the fractional multiplication.

Here is the answer: The problem again was (2 x 5 x 11)/(3 x 5 x 7 x 11); this gives us 5/5 x 11/11 x 2/3 x 1/7 which is equal to 2/(3 x 7).

In frame 17, multiply and reduce the fractions.

Multiplication of Fractions

The answer is 9/11.

In frame 18, multiply and reduce the fractions.

Here is the way the problem would read if you have gotten a correct answer:
12/30 x 10/35 = {(1 x 2 x 2 x 3)/(1 x 2 x 3 x 5)} x {(1 x 2 x 5)/(1 x 5 x 7)}..
This would equal 4/35.

In frame 19, multiply and reduce the fractions 35/42 x 34/55.

The correct answer is 17/33.

Multiply and reduce the fractions in frame 20.

The answer is 2/3.

This concludes the self-instructional tape on the multiplication of fractions. Now you should take the post-test for multiplication of fractions to determine your competency in this area.

Name ____________________

Workbook for Cassette Tape
on
Multiplication of Fractions

1.

$\frac{1}{5} \times \frac{2}{3} = \frac{1 \times 2}{5 \times 3} = \frac{2}{15}$

2.

a) $\frac{1}{7} \times \frac{1}{3} = \frac{1 \times 1}{7 \times 3} =$ __________

b) $\frac{3}{11} \times \frac{3}{5} = \frac{3 \times 3}{11 \times 5} =$ __________

c) $\frac{11}{12} \times \frac{5}{9} = \frac{11 \times 5}{12 \times 9} =$ __________

3.

a) $\frac{3}{8} \times \frac{5}{7} \times \frac{1}{2} = \frac{3 \times 5 \times 1}{8 \times 7 \times 2} =$ __________

b) $\frac{5}{12} \times \frac{7}{11} \times \frac{1}{4} = \frac{5 \times 7 \times 1}{12 \times 11 \times 4} =$ __________

c) $\frac{1}{4} \times \frac{3}{7} \times \frac{5}{13} = \frac{1 \times 3 \times 5}{4 \times 7 \times 13} =$ __________

4.

$\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} \times \frac{g}{h} = \frac{a \times c \times e \times g}{b \times d \times f \times h}$

5.

a) 3 = Prime = 1 x 3

b) 6 = Not Prime = 1 x 2 x 3

c) 7 = Prime = 1 x 7

d) 12 = Not Prime = 1 x 2 x 2 x 3

6.

4 x 3 x 6 = 72

Factors = 4, 3, 6

Product = 72

7.

26 = 2 x 13

27 = 3 x 3 x 3

8.

a) 11 = ________

b) 15 = ________

c) 17 = ________

d) 23 = ________

9.

22 = 1 x 2 x 11 Factors = ________

105 = 1 x 3 x 5 x 7 Products = ________

10.

a) 12 = 2 x 2 x ________

b) 30 = 2 x ________ x ________

c) 25 = ________ x ________

d) 7 = ________ x ________

11.

$$\frac{4}{6} \times \frac{9}{16} \Longrightarrow \frac{2 \times 2}{3 \times 2} \times \frac{3 \times 3}{2 \times 2 \times 2 \times 2}$$

12.

$$\frac{12}{30} \times \frac{10}{35} \Longrightarrow \frac{(\qquad)}{(\qquad)} \times \frac{(\qquad)}{(\qquad)}$$

13.

$$\frac{2 \times 2 \times 3}{2 \times 3 \times 5} = \frac{2}{2} \times \frac{3}{3} \times \frac{2}{5}$$

14.

$$\frac{2 \times 2 \times 3}{2 \times 3 \times 5} = 1 \times 1 \times \frac{2}{5} = \frac{2}{5}$$

15.

a) $1 \times 1 \times 5 = 5$

b) $\frac{2}{2} \times \frac{3}{3} \times \frac{5}{1} = 1 \times 1 \times 5 = 5$

16.

$$\frac{2 \times 5 \times 11}{3 \times 5 \times 7 \times 11} \Longrightarrow \frac{(\qquad)}{(\qquad)} \times \frac{(\qquad)}{(\qquad)} \times \frac{(\qquad)}{(\qquad)} \Longrightarrow \underline{\qquad}$$

17.

$$\frac{3}{5} \times \frac{3}{11} \times \frac{5}{11} \times \frac{11}{1} = \frac{5}{5} \times \frac{11}{11} \times \frac{3 \times 3}{11} = \underline{\qquad\qquad}$$

18.

$$\frac{12}{30} \times \frac{10}{35} = \frac{1 \times 2 \times 2 \times 3}{1 \times 2 \times 3 \times 5} \times \frac{1 \times 2 \times 5}{1 \times 5 \times 7} = \underline{\qquad\qquad}$$

19.

$$\frac{35}{42} \times \frac{34}{55} = \underline{\qquad\qquad}$$

20.

$$\frac{8}{27} \times \frac{70}{16} \times \frac{18}{35} = \underline{\qquad\qquad}$$

Division of Fractions

GENERAL INSTRUCTIONS

1. Before beginning this module, take the pre-test assessment to determine your knowledge for Division of Fractions and their prerequisites.
2. Select one of the two instructional modes -- cassette tape or programmed instruction -- and complete the study module.
3. Take the post-test to determine your mastery of Division of Fractions.

PREREQUISITES

The following prerequisites should have been mastered before undertaking this module.

1. Factoring a Number into Prime Factors
2. Equivalent Fractions
3. Reciprocal of a Number
4. Reduction of Fractions
5. Multiplication of Fractions

PRE AND POST-TEST PERFORMANCE SPECIFICATIONS

Given two different fractions you will divide and record your answer on the assessment instrument. There will be five examples on the instrument. The acceptable performance level will be 80% correct responses.

INSTRUCTIONAL MODES

Choose one of two different instructional modes. Instructional mode number 1 consists of a cassette tape on how to divide a series of fractions. Instructional mode number 2 consists of a series of programmed instructional frames on how to divide a set of fractions.

PROGRAMMED INSTRUCTION
for
DIVISION OF FRACTIONS

Before beginning this module, it is necessary to take the pre-test on Division of Fractions. Programmed instruction is used as a workbook. Each frame contains one or more blanks in which you should write the numbers you feel best complete the statement in the frame.

18 ÷ 3 = _________

6

It is necessary to use some device (a 5" x 8" index card is good) to cover the answer which appears directly below the frame that you are working. It must be pointed out that for maximum benefit, you must not look at the answer before you record your responses in the frame.

Good Luck!

DIVISION OF FRACTIONS

1. Throughout this program, questions you are asked to answer will appear in boxes like this one; these boxes are called frames. The next frame appears below the double line.

The answer will appear in this position on every page.

2. Programmed texts are used, not read. Each frame contains one or more blanks in which you should write the numbers or symbols you feel best complete the statement in the frame.

$12 \div 2 =$ ____________

6

3. As soon as you have written your answer, but not before, look below at the answer.

$1 + 6 - 2 =$ ____________

5

4. Compare your answer with the correct answer. If your answer is the same, proceed to the next frame; if your answer is different, study the frame and the answer until you identify your error. Programs develop progressively and you must understand each frame before you proceed to the next.

$1 \times 2 \times 4 =$ ________

8

5. To introduce division of fractions, consider the following:

$\frac{a}{b} \times \frac{b}{a} = \frac{ab}{ba} = 1 \qquad \frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1 \qquad \frac{5}{7} \times \left(\qquad \right) = 1$

$\frac{7}{5}$

6. $\frac{2}{3} \times \frac{3}{2} = 1$

$\frac{3}{2}$ is called the reciprocal of $\frac{2}{3}$ because $\frac{2}{3} \times \frac{3}{2} =$ ________.

1

7. $\frac{5}{7} \times$ (________) $= 1$

The reciprocal of 5/7 is ________.

$\frac{7}{5}$ for both

8. $\frac{13}{15} \times (\underline{\qquad\qquad}) = 1$

The _________ of 13/15 is _________.

reciprocal, 15/13

10. The reciprocal of 9 is _________.

The product of 9 and its reciprocal = _________.

$\frac{1}{9}$, 1

11. We will see shortly the importance of the reciprocal in the division of fractions.

Division of fractions may be represented in either of these equivalent forms: $\frac{3}{4} \div \frac{5}{7} \Longrightarrow \frac{3/4}{5/7}$ $\qquad \frac{1}{5} \div \frac{4}{11} \Longrightarrow$ _________(Rewrite only).

$\frac{1/5}{4/11}$

12. Recall that multiplying a fractions by $\left(\frac{a}{a}\right)$ or 1 (does/does not) change its value.

does not

13. $\frac{1/5}{3/8}$ (is/is not) equal to $\frac{1/5 \times N}{3/8 \times N}$ since $\frac{N}{N} =$ ________.

is, 1

14. $\frac{1/5}{3/8}$ (is/is not) equal to $\frac{1/5 \times a/b}{3/8 \times a/b}$ because $\frac{a/b}{b/a} =$ ________.

is, 1

15. In $\frac{2/3}{5/7}$ 2/3 is the numerator

5/7 is the denominator

________ is the reciprocal of the denominator.

7/5

16. To solve $\frac{3/5}{3/8}$, multiply numerator and denominator by the reciprocal of the denominator.

$$\frac{2/3}{5/7} = \frac{2/3}{5/7} \times \frac{(\qquad)}{(\qquad)} = \frac{2/3 \times (\qquad)}{(\qquad)}$$

$$\frac{2/3}{5/7} \times \frac{7/5}{7/5} = \frac{2/3 \times 7/5}{1}$$

17. In $\frac{1/2}{3/7}$ __________ is the numerator

__________ is the denominator

__________ is the reciprocal of the denominator.

1/2, 3/7, 7/3

18. $\frac{1/2}{3/7} \times \frac{7/3}{7/3} = \frac{1/2 \times 7/3}{3/7 \times 7/3} = \frac{1/2 \times 7/3}{1}$

Note that multiplying numerator and denominator by the reciprocal of the denominator converts division examples to multiplication problems.

Solve: $\frac{3/7}{2/9} =$ __________

$$\frac{3/7}{2/9} \times \frac{9/2}{9/2} = \frac{3/7 \times 9/2}{1} = 27/14 \text{ or } 1\frac{13}{14}$$

19. To divide 3/4 by 5/7 multiply numerator and denominator by the reciprocal of the denominator.

$$3/4 \div 5/7 \Longrightarrow \frac{3/4}{5/7} \Longrightarrow \frac{3/4 \times 7/5}{5/7 \times 7/5} \Longrightarrow \frac{(\qquad)}{(\qquad)} = \frac{(\qquad)}{(\qquad)}$$

$$\frac{21/20}{1} = 21/20 \text{ or } 1\frac{1}{20}$$

20. $\frac{2/5}{3/7} =$ __________

$$\frac{2/5 \times 7/3}{3/7 \times 7/3} = \frac{14/15}{1} = 14/15$$

21. Divide: 3/7 : 4/5 = ________

$$\frac{3/7}{4/5} \times \frac{5/4}{5/4} = \frac{15/28}{1} = 15/28$$

You have now completed the unit on division of fractions and you should take the post-test to check your competency in this area.

STOP

Tape on
Division of Fractions

We now study the division of fractions beginning with a study of the multiplication of fractions.

To multiply two fractions, you simply multiply the numerators of the fractions together and multiply the denominators together.

In frame 1, 1/5 is multiplied by 2/3, and the result is 2/15.

In frame number 2, multiply the following fractions: a) 1/7 x 1/3 = _____; b) 3/11 x 3/5 = _____; c) 11/12 x 5/9 = _____.

Here are the answers: The answer to a) is 1/21; The answer to b) is 9/55; The answer to c) is 55/108.

In frame number 3, multiply the following fractions: a) 3/8 x 5/7 x 1/2; b) 5/12 x 7/11 x 1/4; and c) 1/4 x 3/7 x 5/13.

Here are the answers: The answer to a) is 15/112; The answer to b) is 35/528; The answer to c) is 15/364.

You have now seen the principle of the multiplication of fractions at work. To multiply fractions, we simply multiply the numerators together and multiply the denominators together.

This principle may be applied to the multiplication of any series of fractions. For example, look at frame 4: a/b x c/d x e/f x g/h = (a x c x e x g)/(b x d x f x h).

The simple steps of changing each term in the numerator and the denominator to prime factors and then reducing will eliminate a great deal of tedious multiplication that often causes errors. You will benefit considerably, therefore, by mastering the approach developed in the following frames.

A prime number is any whole number, other than one, that is divisible only by itself and one.

In frame 5, determine whether the numbers 3, 6, 7, and 12 are prime numbers or not prime numbers.

Here are the answers: a) is a prime number; b) 6 is not a prime number because you can multiply 1 x 2 x 3; c) is a prime number; and 12 is not a prime number because you can multiply 1 x 2 x 2 x 3.

It's seen in that frame that the numbers 6 and 12 can be divided by 1, 2, and 3 and that consequently they are not prime numbers. Two or more numbers multiplied together are termed factors and the result of their multiplication is termed their product.

In frame number 6, we have the following multiplication: 4 x 3 x 6 = 72. The factors are 4, 3, 6 and their product is 72.

Division of Fractions

A number is reduced to its prime factors when it is represented as the product of a series of prime numbers. For example, in frame 7: 26 = 2 x 13; 27 = 3 x 3 x 3. The product of 26 is represented by the multiplication of 2 and 13. These are both prime factors. The product of 27 is represented by 3 x 3 x 3, which are all prime factors.

In frame number 8, identify which of the numbers are prime numbers.

Here are the answers: 11 is a prime number, because 1 x 11 = 11; 15 is not a prime number, because 1 x 3 x 5 = 15; 17 is a prime number, because 1 x 17 =17; 23 is also a prime number because 1 x 23 = 23.

In frame number 9, identify the products and factors of 27 and 105.

Here are the answers: The factors are 1, 2, 11, 3, 5, and 7. The products are 22 and 105.

In frame number 10, rewrite the numbers as prime factors.

Here are the answers: The answer to problem a) 12 = 2 x 2 x 3; problem b) 30 = 2 x 3 x 5; problem c) 25 = 5 x 5; problem d) 7 is already a prime number.

To multiply fractions, first rewrite each number as its prime factors. For example, look at frame number 11: 4/6 x 9/16 = {(2 x 2)/(3 x 2)} x {(3 x 3)/(2 x 2 x 2 x 2)}.

In frame 12, give the factors for 12/30 x 10/35.

Here is the answer: {(2 x 2 x 3)/(2 x 3 x 5)} x {(2 x 5)/(5 x 7)}.

Note that the answer given in 12 is also given in frame 13. And that (2 x 2 x 3)/(2 x 3 x 5) may be regrouped as 2/2 x 3/3 x 2/5. Looking at frame 13 again, since 2/2 equals 1, and 3/3 equals 1,what we have is 1 x 1 x 2/5. Therefore, the result in frame 13 is 2/5. This is clarified for you in frame 14.

The elimination of factors common to numerators and denominators does not change the value of a fraction. The only change is its form. For instance, 2/2 equals 1, 3/3 equals 1, and if you multiply any number by 1, you have a result that is equal to that number. As an example, look at frame 15. Problem a) 1 x 1 x 5 = 5. In problem b) 2/2 x 3/3 x 5/1 = 1 x 1 x 5 = 5. As it is seen, the value is not changed. The elimination of factors common to both the numerator and denominator of a fraction is termed reducing.

In frame 16, reduce the fractional multiplication.

Here is the answer: The problem again was (2 x 5 x 11)/(3 x 5 x 7 x 11); this gives us 5/5 x 11/11 x 2/3 x 1/7 which is equal to 2/(3 x 7).

In frame 17, multiply and reduce the fractions.

Division of Fractions

The answer is 9/11.

In frame 18, multiply and reduce the fractions.

Here is the way the problem would read if you have gotten a correct answer: 12/30 x 10/35 = {(1 x 2 x 2 x 3)/(1 x 2 x 3 x 5)} x {(1 x 2 x 5)/(1 x 5 x 7)}. This would equal 4/35.

In frame 19, multiply and reduce the fractions 35/42 x 34/55.

The correct answer is 17/33.

Multiply and reduce the fractions in frame 20.

The answer is 2/3.

It was necessary to review the multiplication of fractions before we can learn how to divide fractions. In division of fractions, you use the same techniques as we learned in the multiplication of fractions, except that you encounter the concept of reciprocal.

The reciprocal of a number is defined as that number that you can multiply a number by and get an answer of 1. For example, look at frame 21: 1/2 x 2 = 1; 2/3 x 3/2 = 1; and 3/7 x 7/3 = 1. It is seen that 1/2 is the reciprocal of 2, 2/3 is the reciprocal of 3/2, and 3/7 is the reciprocal of 7/3. It's rather obvious that to get the reciprocal of a number we simply turn it upside down.

In frame 22, give the reciprocals for the numbers indicated.

Here are the answers: The reciprocal of 3/5 is 5/3; the reciprocal of 7/11 is is 11/7; the reciprocal of 5 is 1/5; the reciprocal of 4/3 is 3/4.

In the division of fractions, you simply change the divisor to its reciprocal and multiply. For example, look at frame 23. 5 ÷ 1/3 equals 5 times the reciprocal of 1/3 or 3/1, resulting in an answer of 15.

In frame 24, find the quotient of the following fractions. The term quotient merely means the answer to a division problem.

The answers are: Problem a) 2/15; Problem b) 9/7

In frame 25, divide and reduce the following fractions.

Here are the answers, as we look at the problems together: For problem a) 12/35 ÷ 4/7 = 3/5; for problem b) 33/13 ÷ 5/39 = 99/5.

In some cases, division of fractions is written as shown in frame 26. This type of division is better written in the form shown in frame 27.

By rewriting the fraction as shown in frame 27, it puts it in a form that will lead to fewer errors. In frame 28, do the division of the fractions.

Division of Fractions

The correct answer is 5/3.

In frame 29, divide and reduce the fractions {34/39} ÷ {17/3}.

The correct answer is 2/13.

Now, divide and reduce the fractions in frame 30.

The correct answer is 7/5.

This concludes our self-study of the division of fractions. Now, please take the post-test to help determine your competency in this area.

Name ________________

Workbook for Cassette Tape
on
Division of Fractions

1.

$\frac{1}{5} \times \frac{2}{3} = \frac{1 \times 2}{5 \times 3} = \frac{2}{15}$

2.

a) $\frac{1}{7} \times \frac{1}{3} = \frac{1 \times 1}{7 \times 3} =$ ________

b) $\frac{3}{11} \times \frac{3}{5} = \frac{3 \times 3}{11 \times 5} =$ ________

c) $\frac{11}{12} \times \frac{5}{9} = \frac{11 \times 5}{12 \times 9} =$ ________

3.

a) $\frac{3}{8} \times \frac{5}{7} \times \frac{1}{2} = \frac{3 \times 5 \times 1}{8 \times 7 \times 2} =$ ________

b) $\frac{5}{12} \times \frac{7}{11} \times \frac{1}{4} = \frac{5 \times 7 \times 1}{12 \times 11 \times 4} =$ ________

c) $\frac{1}{4} \times \frac{3}{7} \times \frac{5}{13} = \frac{1 \times 3 \times 5}{4 \times 7 \times 13} =$ ________

4.

$\frac{a}{b} \times \frac{c}{d} \times \frac{e}{f} \times \frac{g}{h} = \frac{a \times c \times e \times g}{b \times d \times f \times h}$

5.

a) 3 = Prime = 1 x 3

b) 6 = Not Prime = 1 x 2 x 3

c) 7 = Prime = 1 x 7

d) 12 = Not Prime = 1 x 2 x 2 x 3

6.

4 x 3 x 6 = 72

Factors = 4, 3, 6

Product = 72

7.

26 = 2 x 13

27 = 3 x 3 x 3

8.

a) 11 = ________

b) 15 = ________

c) 17 = ________

d) 23 = ________

9.

22 = 1 x 2 x 11 Factors = ________

105 = 1 x 3 x 5 x 7 Products = ________

10.

a) 12 = 2 x 2 x ________

b) 30 = 2 x ________ x ________

c) 25 = ________ x ________

d) 7 = ________ x ________

11.

$$\frac{4}{6} \times \frac{9}{16} \Longrightarrow \frac{2 \times 2}{3 \times 2} \times \frac{3 \times 3}{2 \times 2 \times 2 \times 2}$$

12.

$$\frac{12}{30} \times \frac{\text{[illegible]}2}{\text{[illegible]}5} \Longrightarrow \frac{(\qquad\qquad)}{(\qquad\qquad)} \times \frac{(\qquad\qquad)}{(\qquad\qquad)}$$

13.

$$\frac{2 \times 2 \times 3}{2 \times 3 \times 5} = \frac{2}{2} \times \frac{3}{3} \times \frac{2}{5}$$

14.

$$\frac{2 \times 2 \times 3}{2 \times 3 \times 5} = 1 \times 1 \times \frac{2}{5} = \frac{2}{5}$$

15.

a) $1 \times 1 \times 5 = 5$

b) $\frac{2}{2} \times \frac{3}{3} \times \frac{5}{1} = 1 \times 1 \times 5 = 5$

16.

$$\frac{2 \times 5 \times 11}{3 \times 5 \times 7 \times 11} \Longrightarrow \frac{(\qquad)}{(\qquad)} \times \frac{(\qquad)}{(\qquad)} \times \frac{(\qquad)}{(\qquad)}$$

17.

$\frac{3}{5} \times \frac{3}{11} \times \frac{5}{11} \times \frac{11}{1} = \frac{5}{5} \times \frac{11}{11} \times \frac{3 \times 3}{11} =$ __________

18.

$\frac{12}{30} \times \frac{10}{35} = \frac{1 \times 2 \times 2 \times 3}{1 \times 2 \times 3 \times 5} \times \frac{1 \times 2 \times 5}{1 \times 5 \times 7} =$ __________

19.

$\frac{35}{42} \times \frac{34}{55} =$ __________

20.

$\frac{8}{27} \times \frac{70}{16} \times \frac{18}{35} =$ __________

21.

a) $\frac{1}{2} \times 2 = 1$

c) $\frac{2}{3} \times \frac{3}{2} = 1$

d) $\frac{3}{7} \times \frac{7}{3} = 1$

22.

a) $\frac{3}{5} =$ __________ c) $\frac{5}{1} =$ __________

b) $\frac{7}{11} =$ __________ d) $\frac{4}{3} =$ __________

23. $5 \div \frac{1}{3} = 5 \times \frac{3}{1} = \frac{15}{1}$

24.
a) $\frac{2}{3} \div \frac{5}{1} =$ ______
b) $\frac{3}{7} \div \frac{1}{3} =$ ______

25.
a) $\frac{12}{35} \div \frac{4}{7} =$ ______
b) $\frac{33}{13} \div \frac{5}{39} =$ ______

26. $\frac{3/7}{8/17}$

27. $\frac{3/7}{8/17} = \frac{3}{7} \div \frac{8}{17}$

28. $\frac{5/13}{3/13} = \frac{5}{13} \div \frac{3}{13} =$ ______

29. $\frac{34/39}{17/3} =$ ______

30. $\frac{12/30}{10/35} =$ ______

PRE-TEST: OPERATIONS ON FRACTIONS MINI-MODULES
(LCD, Addition and Subtraction)

DIRECTIONS: In the following examples you will encounter problems dealing with operations of addition and subtraction of fractions. Please answer each problem carefully.

1. Find the Least Common Multiple of the following numbers: 3, 4, and 5.

ANSWER ____________________

2. Identify which of the following numbers are prime.

EXAMPLE: a) 3 = prime
b) 9 = __________
c) 26 = __________
d) 29 = __________

3. Completely factor the following numbers:

EXAMPLE: a) 18 = 3 x 3 x 2
b) 30 = __________
c) 63 = __________

4. Find the Least Common Denominator of the following fractions:

EXAMPLE: a) $\frac{3}{17} + \frac{7}{34} + \frac{11}{68}$, LCD = 68
b) $\frac{3}{13} + \frac{7}{26} + \frac{5}{39}$, LCD = __________
c) $\frac{2}{11} + \frac{5}{66} + \frac{2}{77}$, LCD = __________

5. Add or subtract the following fractions:

a) $\frac{3}{13} + \frac{7}{26} + \frac{5}{39} =$ __________
b) $\frac{2}{11} + \frac{5}{66} + \frac{2}{77} =$ __________
c) $\frac{1}{2} - \frac{1}{3} - \frac{1}{7} =$ __________
d) $\frac{2}{11} - \frac{5}{66} - \frac{2}{77} =$ __________

PRE-TEST: OPERATIONS ON FRACTIONS MINI-MODULES
(Multiplication and Division)

DIRECTIONS: In the following examples you will encounter problems dealing with operations of multiplication and division of fractions. Please answer each problem carefully.

1. Identify which of the following numbers are prime:

Example: a) 3 = Prime
b) 9 = ________
c) 26 = ________
d) 29 = ________

2. Completely factor the following numbers:

Example: a) 18 = 3 x 3 x 2
b) 30 = ________
c) 63 = ________

3. Give the reciprocal of the following numbers:

Example: a) 3 = 1/3
b) 7/8 = ________
c) 7/2 = ________

4. Reduce the following fractions:

Example: a) $\frac{26}{39} = \frac{2 \times 13}{3 \times 13} = \frac{2}{3} \times \frac{\cancel{13}}{\cancel{13}} = \frac{2}{3}$

b) $\frac{16}{112} =$

c) $\frac{32}{56} =$

5. Multiply or divide the following fractions:

a) $\frac{3}{17} \times \frac{51}{3} =$ ________

b) $\frac{7}{13} \times \frac{39}{49} =$ ________

c) $\frac{5}{26} : \frac{25}{39} =$ ________

d) $\frac{7/22}{49/11} =$ ________

POST-TEST: OPERATIONS ON FRACTIONS MINI-MODULES
(LCD, Addition and Subtraction)

DIRECTIONS: In the following examples you will encounter problems dealing with operations of addition and subtraction of fractions. Please answer each problem carefully.

1. Find the Least Common Multiple of the following numbers: 5, 7 and 35.

   ANSWER __________

2. Identify which of the following numbers are prime.

   EXAMPLE: a) 3 = prime
   b) 7 = __________
   c) 21 = __________
   d) 31 = __________

3. Completely factor the following numbers:

   EXAMPLE: a) 18 = 3 x 3 x 2
   b) 40 = __________
   c) 102 = __________

4. Find the Least Common Denominator of the following fractions:

   EXAMPLE: a) $\frac{3}{17} + \frac{7}{34} + \frac{11}{68}$, LCD = 68
   b) $\frac{1}{13} + \frac{4}{39} + \frac{2}{68}$, LCD = __________
   c) $\frac{1}{5} + \frac{3}{20} + \frac{7}{100}$, LCD = __________

5. Add or subtract the following fractions:

   a) $\frac{2}{11} + \frac{7}{22} + \frac{3}{88} =$ __________
   b) $\frac{3}{7} + \frac{2}{17} + \frac{7}{119} =$ __________
   c) $\frac{12}{13} - \frac{2}{78} - \frac{2}{117} =$ __________
   d) $\frac{5}{8} - \frac{3}{64} - \frac{9}{192} =$ __________

POST-TEST: OPERATIONS ON FRACTIONS MINI-MODULES
(Multiplication and Division)

1. Identify which of the following numbers are prime.

Example: a) 3 = Prime
b) 11 = __________
c) 68 = __________
d) 31 = __________

2. Completely factor the following numbers:

Example: a) 18 = 3 x 3 x 2
b) 68 = __________
c) 76 = __________

3. Give the reciprocal of the following numbers:

Example: a) 3 = 1/3
b) 5/3 = __________
c) 13/2 = __________

4. Reduce the following fractions:

Example: a) $\frac{26}{39} = \frac{2 \times 13}{3 \times 13} = \frac{2}{3} \times \frac{13}{13} = \frac{2}{3}$
b) $\frac{32}{160} =$ __________
c) $\frac{72}{182} =$ __________

5. Multiply or divide the following fractions:

a) $\frac{22}{36} \times \frac{21}{11} =$ __________
b) $\frac{17}{33} \times \frac{26}{51} =$ __________
c) $\frac{7}{30} \div \frac{21}{10} =$ __________
d) $\frac{17/26}{68/52} =$ __________

APPENDIX E

Curriculum Guides

In the teaching of courses for any subject area there is a need for the instructor to know which topics should be taught and how the student is expected to demonstrate his knowledge of these topics. According to Taba (1962) a curriculum provides a "plan for learning".

This section is composed of a generalized curriculum guide which outlines the concepts selected by jury members as necessary and observable behaviors that a student is expected to exhibit in demonstrating his mastery of each concept.

During the period of jury validation of the rating instrument, the staff asked the instructors to give their thoughts about curriculum guides. Most jurors agreed that an outline of topics was the most important item. They further expressed the desire to see a definite statement concerning exactly what performance was to be expected of the student.

With this feedback in mind, several curriculum experts were consulted to determine the best approach for the guides. The experienced curriculum designers were in agreement with what the jurors had said. The consultants also recommended that since the guides were to be designed for use in several institutions, there would be greater flexibility if no specific supplementary materials were referenced. This exclusion would allow each vocational school to use its current texts and the filmstrips it had available.

For each concept, there is a wide variety of approaches to be used by an instructor. These include class lectures or demonstrations, filmstrips, programmed instruction modules, cassette tapes, and peer group help sessions.

BASIC ARITHMETIC

Real Numbers

Concept	Behavioral Objective
1. Basic operations with whole numbers:	Given a set of five typical problems representing the operations of addition, subtraction, multiplication and division, the student will be able to correctly solve eighty percent of the problems.
2. Combined operations with whole numbers:	Using representative problems with various combinations of addition, subtraction, multiplication and/or division the student will illustrate his ability to work problems by showing necessary steps for obtaining correct solutions of 80% of the problems given.
3. Definition of signed numbers:	The student will be able to correctly state the definition of a signed number.
4. Graphing signed numbers:	The student will demonstrate his understanding of signed numbers by using a number line to correctly lable & identify the relative values of a set of positive, negative and zero integers with an accuracy of 80%.
5. Basic operations with signed:	The student will solve a set of five representative problems to demonstrate his knowledge of the laws governing operations with signed numbers. Competency level must be at least 80% to be acceptable.

Concept	Behavioral Objective
6. Sets:	Five lists which include both numbers or objects, the student will sort the ~~number~~ of object into groups (sets). Competency is demonstrated by 80% correct grouping.
7. One-to-one correspondence:	Given two sets of objects the student will set up a one-to-one correspondence between the elements of the two sets. Example one-to-one correspondence between natural numbers and intervals on a number line.
8. Rounding off:	Given five decimal numbers and five common fractions, the student will round off these to the nearest whole number. When given five whole numbers, the student will round off to the nearest tenth. A competency level of 80% on each group is required.
9. Prime numbers:	Given a list of 10 numbers, the student will correctly identify at least 8 as prime.
10. Composite numbers:	Given a list of 10 numbers, the student will identify at least 8 as composite.
11. Greatest common divisor:	Given five lists of fractions, the student will correctly identify the Greatest Common Divisor for each set. Competency level is 80%.
12. Least common multiple:	Given five lists of fractions, the student will identify the Least Common Multiple for each list. Competency is demonstrated by 80% correct identification.

Real Numbers

Concept	Behavioral Objective
13. Absolute value:	The student will state the definition of the absolute value of a number.

Common Fractions

Concept	Behavioral Objective
14. Reading & writing common fractions:	The student will state the definition of a common fraction.
15. Reading a ruler:	Given ten lines of varying length, the student will determine the correct length of each by using a ruler. He should achieve a competency level of 80%
16. Basic operation with fractions:	The student will demonstrate his abilities to add, subtract, multiply and divide by solving, to 80% competency, five each of fractional probelms representing the basic operations.
17. Equivalent fractions:	Given a listing of 15 fractions, the student will correctly identify the 10 equivalent fractions. Competency is achieved by 80% accuracy.
18. Converting mixed numbers to improper fractions:	Given five mixed numbers, the student will convert at least four to improper fractions.
19. Basic operations with mixed numbers:	The student will demonstrate his competency by adding, subtracting, multiplying and dividing at least eight out ten representative examples of mixed numbers.

Common Fractions

Concept	Behavioral Objective
20. Combined operations with mixed fractions:	For a given set of problems involving combined operations with mixed fractions, the student will correctly solve four out of five problems.
21. Proper and improper fractions:	From a list of ten fractions, the student will identify each as either proper or imporper. He will then reduce improper fractions. Minimum performance level is eight correct answers.
22. Complex numbers:	The student will state the definition of a complex number and will be able to correctly identify which numbers in a list are complex in four of five examples.
23. Combining like terms:	The student will demonstrate his ability to combine like terms by correctly solving eight of ten problems involving common fractions with like denominators.
24. Reducing to lowest terms:	Given a set of ten common fractions, the student will reduce at least eight to lowest terms by cancelling common fractions.
25. Finding the lowest common denominator:	The student will be able to find the LCD of a group of fractions by factoring each denominator into prime numbers, then combining the necessary factors.
26. Basic operations with common fractions and mixed fractions:	From a set of ten probelms involving common and mixed fractions, the student will demonstrate his mastery of this concept by performing basic operation to solve at least eight problems.

Decimal Fractions

Concept	Behavioral Objective
27. Reading and writing:	From a list of decimal fractions the student will be able to correctly read four out of five values. The student will be able to write with 80% accuracy the decimal fractions that are dictated to him.
28. Rounding off:	The student will be able to round off four out of five given decimal fractions.
29. Basic operations:	The student will demonstrate his competency by correctly working five each of problems involving addition, subtraction, multiplication and division of decimal fractions.
30. Table of decimal equivalents:	Given a list of ten fractions and ten decimal fractions, the student will use a table of decimal equivalents to find the equivalent to the given lists. Competency level is 80%.
31. Fraction to decimal conversion:	Given a list of 10 fractions, the student will correctly convert at least eight of these to decimal fractions, by dividing the denominator into the numerator.
32. Decimal to fraction conversion:	Given a list of ten decimal fractions, the student will convert at least 80% of these into common fractions.

Scientific Notation

Concept	Behavioral Objective
33. Concept of powers of 10:	The student will be able to correctly explain the use of powers of ten notation.

Scientific Notation

Concept	Behavioral Objective
34. Significant figures:	Given a set of ten values, the student will identify the number of significant figures in at least eight.
35. Rounding off:	From a list the student will accurately round of values to a specified number of digits with at least 80% correct responses.
36. Basic operations using powers of 10:	The student will demonstrate his ability to perform basic opwrations with powers of ten notation by finding the solution of eight to ten problems involving addition, subtraction, multiplication and division with powers of ten notation.
37. Raising to powers using powers of 10:	From a given list of values, the student will correctly raise 80% of the values to a specified power using powers of 10 notation.
38. Extracting square roots using powers of 10:	The student will demonstrate his ability to extract square roots using powers of ten by solving eight of ten assigned problems.

Squares and Square Roots

Concept	Behavioral Objective
39. Using the table of square roots:	Given a list of 10 numbers, the student will correctly find the square roots of the number from a table of powers and roots. The competency level is 80%.

Squares and Square Roots

Concept	Behavioral Objective
40. Finding the square of a number:	The student will correctly evaluate eight of ten given expressions illustrating squares.
41. Finding the square root of a number:	Given a list of ten numbers, the student will correctly find the square roots of at least eight.
42. Finding the square root by the long division method:	Given a list of ten numbers, the student will correctly find the square root of at least eight using long division.
43. Basic operations of radicals:	Given ten each of problems involving subtraction, addition, multiplication and division of radicals, the student will correctly solve 80% of the problems.
44. Radical sign and index of roots:	Given a set of radical expressions, the student will identify with at least an 80% accuracy the index number, and the sign of the specified.
45. Pythagorean theorem:	The student will correctly state the Pythagorean theorem and will tell how it is used.

Percentage

Concept	Behavioral Objective
46. Definition:	The student will be able to correctly state the definition of percentage.
47. Converting fractions to percent:	Given a series of ten fractions, the student will correctly convert at least eight to percents.

Percentage

Concept	Behavioral Objective
48. Converting percents to fractions:	Given a list of ten percents, the student will correctly convert at least eight to fractions.
49. Converting decimals to percent:	Given a list of ten decimal fractions, the student will correctly convert at least eight to percents.
50. Converting percents to decimals:	Given a list of ten percents, the student will correctly convert at least eight to decimal fractions.
51. Finding the percentage given the base and rate:	Given ten problems involving the base and rate, the student will correctly compute the percentage for each. Competency level is 80%.
52. Finding the rate given the precent and base:	Given ten probelms involving the percentage and base, the student will correctly compute the rate for each Competency level is 80%.
53. Finding the base given the rate and percentage:	Given ten probelms involving the rate and percentage, the student will correctly compute the base for each. Competency level is 80%
54. Solving problems:	Given a set of problems related to his vocational area, the student will make calculations involving base, rate, and percentage to solve at least four our of five problems.
55. Price lists and discount:	From a given list the student will be able to read prices and calculate purchase discounts of items with at least 80% accuracy.

Percentage

Concept	Behavioral Objective
56. Percentage error and averaging:	With a minimum performance level of eight out of ten correct responses, the student will be able to calculate percentage error and find averages for a given set of problems.
57. Markup:	Given a list of ten prices and a markup rate, correctly determine the acount of markup for at least eight of the items listed.

Ratio and Proportion

Concept	Behavioral Objective
58. Definition of ratio:	The student will be able to correctly state the definition of ratio.
59. Reduction of ratio to lowest terms:	Given a list of ten ratios, the student will be able to correctly reduce at least eight to lowest terms.
60. Definition of porportion:	The student will be able to correctly state the devinition of proportion.
61. Finding the unkown term in a proportion:	Given a proportion with an unknown term the student will solve the problem to determine its value.
62. Application of porportion:	From a set or probelms related to his vocational area, the student will set up the appropriate proportions and find their solutions with 80% accuracy.

Concept	Behavioral Objective
63. The principle of levels:	The student will define a lever and will describe ways in which levers are used in his vocational area.
64. Principle of pulleys:	The student will define a pulley and will describe ways in which pulleys are used in his vocational area.
65. Principle of gears:	The student will define a gear and will describe ways in which gears are used in his vocational area.
66. Principle of hydraulic press:	The student will define a hydraulic press and will describe ways in which a hydraulic press is used in his vocational area.
67. The English system of weights and measures:	The student will convert English weights and measures with a minimum performance level of 80% accuracy.
68. The metric of weights and measures:	The student will convert weights and measures within the metric system with at least 80% accuracy.
69. The English-metric conversion:	Given a set of values, the student will convert metric values to the equivalent English values, and English values to the equivalent metric values with 80% accuracy.
70. The binary number system:	The student will correctly state the definition of a binary number.
71. The computer number system:	From a list of ten numbers, the student will correctly sort at least eight according to whether or not they are in a form used by computers.

Ratio and Proportion

Concept	Behavioral Objective
72. Computing like a computer:	Given a set of binary numbers the student will correctly sort at least eight according to whether or not they are in a form used by computers.
73. Binary to decimal conversion:	The student will convert four of five binary numbers to their decimal equivalents.
74. Decimal to binary conversion:	The student will convert four of five decimal numbers to their binary equivalents.
75. Basic operations of binary numbers:	The student will correctly solve eight of ten problems involving basic operations with binary numbers.
76. Multiplying and dividing measurements:	The student will demonstrate his ability to multiply and divide measurements by correctly solving at least 80% of a representative set of problems.
77. Table of equivalent measurements:	For a set of given measurements, the student will be able to find equivalent values from a table with a minimum level of 80% correct responses.

Instruments of Measuring

Concept	Behavioral Objective
78. Linear scales:	The student will use rulers to draw lines of lengths specified in fractional units or meter units. Fractional units must be accuracy to within 1/16".

Concept	Behavioral Objective
79. Vernier calipers:	The student will use vernier calipers to measure the inside and outside diameters of a piece of pipe to within ± 1/64".
80. Micrometers:	The student will use the micrometer to measure the diameter of a section of pipe to within .001".
81. The protractor:	From drawings representing acute and obtuse angles, the student will measure the number of degrees with the use of a protractor. Minimum performance level will be measurement within ± 2°.
82. The planimeter:	The student will determine the area of four of five figures by tracing their edge with a planimeter.
83. The guage number of wire:	The student will demonstrate an ability to determine the guage number of wire by using a measurement instrument to correctly identify the guage of five pieces of wire.
84. The framing square:	The student will describe the methods for using the framing square.
85. The tachometer:	The student will explain the scale and uses of the tachometer.
86. Angular scales:	The student will be able to explain what is meant by angular scales & to tell when they can be used.

Instruments of Measuring

Concept	Behavioral Objective
87. The circumference rule:	The student will be able to define "circumference" and will state the formula for finding circumference.

Length

Concept	Behavioral Objective
88. Basic operations using length:	With the use of both English and metric rulers, the student will measure the length of lines. Required accuracy will be within 1/4" English or .1 cm metric measure.
89. Length as a measure of distance:	The student will be able to state the definition of length as a measure of distance.
90. Calculations of a rectangular (perimeter, width, length):	From a set of drawings, the student will measure length & width within 1/4" of actual value, and will use his measures to find the perimeter of at least 80% of the drawings.
91. Calculations of a triangle:	The student will measure the length of the isdes and the altitude of a given set of triangles and will use the measurements to calculate the area & perimeter of four out of five given triangles.
92. Calculations of a circle:	From given drawings, the student will use a ruler to measure the diamter within 1 cm. He will then calculate the radius & circumference with in 80% accuracy.

Length

Concept	Behavioral Objective
93. Finding the diameter of a circle when the area is given:	Using the formula for area of a circle, the student will calculate the diameter for four of five circles for which the area is known.
94. The perimeter of the ellipse:	Using available information, the student will calculate the perimeter of a set of ellipses with 80% accuracy.
95. The perimeter of polygons:	The student will calculate the perimeter of a set of polygons with the use of a ruler to within 1/4" of actual value.

Area

Concept	Behavioral Objective
96. As a measure of surface space:	The student will explain the definition of area as used to measure surface space.
97. Calculations of four-sided firuges (parallelogram, trapezoid, square, and rectangle):	With minimum performance level of 80% correct answers, the student will calculate the areas of a representative set of four-sided figures.
98. Area calculations for a circle:	Using the formula for the area of a circle, the student will find the area for four out of five circles for which he has been given either.
99. Relationship of radius, diamter, circumference arcs, sectors, chords, and tangents:	The student will define and distinguish between the component parts of a circle.

Area

Concept	Behavioral Objective
100. The properties of π:	The student will state the properties for π.
101. Area of a regular hexagon:	The student will state the formula for finding the area of a regular hexagon and will find the area of a given hexagon.
102. Regular polygon:	The student will define what is meant by regular polygon and will identify four of five drawings correctly according to whether or not they fit the definition.

Volume

Concept	Behavioral Objective
103. Measures of volume:	The student will name at least three measures of volume in the metric and in the English system and will give the relationships existing between the three.
104. Volume of rectangular solids (cubes):	Given the dimensions of five cubes, the student will find the volume of at least four.
105. Volume of a sphere:	Given the dimensions of five spheres the student will find the volume of at least four.
106. Segments and zones of spheres:	The student will define the terms segment and zone of a sphere.
107. Volume of a cylinder:	Given the dimensions of five cylinders, the student will find the volume of at least four.

Volume

Concept	Behavioral Objective
108. Surface volume of solid and hollow cylinders:	The student will define the term surface volume and explain how it is used for hollow & solid cylinders.
109. Heights of cylinders:	Givne five representative problems, the student will find the height of at least four.
110. Lateral surface of cylinders:	The student will define the term lateral surface of a cylinder:
111. Volume of compound figures:	Given the dimension of five compound figures, the student will find the volume of at least four.
112. Volume of composite figures:	Given the dimensions of five composite figures, the student will find the volume of at least four.

Weight

Concept	Behavioral Objective
113. Measures of mass:	The student will give the units of mass measurement.
114. Specific gravity:	The student will define specific gravity.
115. Density:	The student will define density.
116. Weight standards:	The student will explain the various weight standards.
117. Weights of materials:	The student will find the weight of five various materials with 80% accuracy.

Weight

Concept	Behavioral Objective
118. Weight of castings from patterns:	The student will read a pattern and determine the weight of a casting.

Time

Concept	Behavioral Objective
119. Units of time:	The student will be able to convert from one time unit to another. Minimum proficiency will be converting four out of five values to their equivalents values in a different specified unit.
120. Distance in relation to time:	The student will explain the similarity of distance and time.
121. Frequency:	The student will state the definition of frequency and give at least one example of its use.
122. Period:	The student will state the definition of period and give at least one example of its use in his vocational area.

Money Calculations

Concept	Behavioral Objective
123. Price computation:	The student will determine the price computation of five items and services with 80% accuracy.

Money Calculations

Concept	Behavioral Objective
124. Income computation:	The student will explain how to make an income computation.
125. Interest computation:	The student will calculation the interest on five problems giving the interest rate and time.

Scale Drawings

Concept	Behavioral Objective
126. Finding the drawing measure:	From a scaled drawing the student will use the legend to determine the drawing measure.
127. Finding the actual dimensions of an object from the measured dimension of a diagram:	Given a diagram, the student will explain how to find the dimensions of objects.
128. Finding the area of a figure drawn to a certain scale:	The student will explain how to find the area of a figure drawn to scale.

Slide Rule

Concept	Behavioral Objective
129. Reading the C and D scales:	Given five values the student will use a slide rule and locate at least four of five values on the C & D scale.
130. Slide rule operations (basic):	The student will state the basic operations on a slide rule with 80% accuracy.
131. Laws of exponents:	The student will state the laws of exponents with 80% accuracy.

Slide Rule

Concept	Behavioral Objective
132. Reciprocal, squares and square roots:	The student will demonstrate how to find reciprocals, squares and square roots on a slide rule with 80% accuracy.
133. Cubes and cube roots:	The student will find four out of five cubes and cube roots on a slide rule.
134. Locating the decimal point:	The student will explain how to find the decimal point in slide rule operations.
135. Combination problems:	The student will work four out of five combination problems on a slide rule.
136. Extracting roots of numbers and raising powers:	The student will explain how to extract a root and raise a power on a slide rule.
137. Multiplying and dividing numbers by π:	The student will demonstrate the operations of multiplying and dividing numbers by π on a slide rule.
138. The reciprocal scales:	The student will explain the use of the reciprocal scale with 80% accuracy.
139. The tangent scale:	The student will demonstrate the use of the tangent scale.
140. The ARC tangent of a number:	The student will demonstrate how to find the arc tangent of a number.
141. The sine scale:	The student will explain the sine scale.
142. The ARC sine of a number:	The student will demonstrate how to find the arc sine of a number.
143. The ST scale:	The student will explain the use of the ST scale.

Slide Rule

Concept	Behavioral Objective
144. The cosine:	The student will explain the cosine scale.
145. The cotangent:	The student will explain the cotangent scale.
146. Proportion scale:	The student will demonstrate the use of the proportion scale:
147. Finding the circumference and area of a circle:	Given five probelms, the student will find the circumference and area of at least four.

Geometrical Construction

Concept	Behavioral Objective
148. Bisect or find mid-point of a line:	The student will explain how to bisect a line and identify what is meant by mid-point.
149. Bisect an angle:	The student will bisect an angle with 80% accuracy.
150. Bisect an arc:	The student will bisect an arc with 80% accuracy.
151. Erect a perpendicular to a line at a given point on the line:	he student will erect a perpendicular to a line at a given point.
152. Erect a perpendicular to a line at the end of the line:	The student will erect a perpendicular to a line at the end of a line.
153. Erect a perpendicular through a point not on the line:	The student will erect a perpendicular through a point not on the line.
154. Draw a line through given points parallel to a given line:	The student will draw a line through given points parallel to a given line.

Geometrical Contruction

Concept	Behavioral Objective
155. Divide a given line into a number of equal parts:	The student will divide a given line into a number of equal parts.
156. Draw an angle equal to a given angle:	The student will draw an angle equivalent to a given angle.
157. Draw an equilateral triangle of given size:	The student will draw an equilateral triangle to a given scale.
158. Draw a circle through three given points:	The student will draw a ircle through three given points.
159. Find the center of a circle or circular arc.:	The student will demonstrate how to find the center of a circular arc.
160. Inscribe a square in a circle:	The student will inscribe a square in a circle.
161. Construct a square of given size:	The student will construct a square of a given scale.
162. Inscribe a hexagon in a circle:	The student will inscribe a hexagon in a circle.
163. Inscribe a equilateral triangle in a circle:	The student will inscribe a equilateral triangle in a circle.
164. Construct a hexagon with one of the sides on a given line:	The student will construct a hexagon with one side on a given line.
165. Draw a tangent to a circle at a point on the circumference:	The student will construct a tangent to a circle at a given point on the circumference.
166. Draw a tangent to a circle through a point outside the circle:	The student will construct a tangent to a circle through a point outside the circle.

Geometrical Construction

Concept	Behavioral Objective
167. Draw a tangent to two circles of equal size:	The student will construct a tangent to two circles of equal radii.
168. Draw an internal tangent to two circles of equal size:	The student will construct an internal tangent to two circles of euqal radii.
169. Draw an external tangent to two circles of unequal size:	The student will construct an external tangent to two circles of unequal radii.
170. Draw an internal tangent to two unequal circles:	The student will construct an internal tangent to two circles of unequal radii.

BASIC ALGEBRA

General Numbers

Concept	Behavioral Objective
1. Properties of real numbers:	The student will correctly list the properties of real numbers and will give a numeric example of each.
2. Substitution of variables into algebraic expressions:	The student will correctly solve eight out of ten problems from a given set by substituting variables into algebraic expressions.
3. Translation of word expressions into algebraic expressions:	Given a set of word expressions, the student will convert these to equivalent algebraic expressions. Minimum competency will be demonstrated with the correct responses of 80% of the problems.
4. Order of operations:	Given a set of five problems, the student will demonstrate his knowledge of the concept by performing the operations in the proper sequence to finding the correct solution to at least four of the problems.
5. Evaluating expressions containing variables:	The student will solve at least four out of five expressions to determine the correct value for the variable involved.
6. Basic operations involving variables:	The student will demonstrate his ability to perform basic operations with variables by correctly solving four out of five representative problems.

General Numbers

Concepts	Behavioral Objective
7. Constants and variables:	From a list of ten expressions, the student will correctly identify constant terms from variable terms in at least eight expressions.
8. Positive and negative numbers:	The student will find the correct solution to 80% of a given set of problems representative of the various operations involving signed numbers.
9. Powers of exponents:	Given a set of five problems, the student will correctly identify the exponent and base involved and evaluate the expression. Minimum accuracy will be four correct answers.
10. Absolute value:	Given a set of five values, the student will give the absolute value to at least four.
11. Formulas:	The student will be able to state the formulas required for using powers of exponents.
12. Ratio & porportion:	The student will demonstrate an understanding of ratio & proportion by finding the value of the unknown in 80% of the given problems.
13. Axioms:	The student will be able to identify the axioms from a given list of statements with 80% accuracy.

Arithmetic Operations with Algebraic Expressions

Concept	Behavioral Objective
14. Basic operations with algebraic numbers:	Given a representative set of problems involving algebraic numbers, the student will correctly solve at least four out of five problems.
15. Operations with signs of grouping:	The student will solve a set of problems containing signs of grouping by showing each step involved in removing the signs in order to achieve the correct solution. Minimum performance level will be 80% accuracy.
16. The distributive law:	Given a set of five problems, the student will demonstrate his ability to apply the distributive law by correctly solving at least four of the problems.
17. Equations involving parentheses:	The student will show his ability to work equations involving parenthesis by showing the steps needed to solve four of five representative problems containing parentheses.
18. First degree equations with signed numbers:	The student will find the correct answer to four of five representative problems involving signed numbers.
19. Adding & subtracting like terms:	The student will correctly solve a representative set of problems by adding/subtracting like terms. Minimum acceptable performance will be 80% correct responses.

Arithmetic Operations with Algebraic Expressions

Concept	Behavioral Objective
20. Multiplication & division of monomials and polynomials:	From a representative group of five problems, the student will find the correct solutions to at least four by applying the rules governing multiplication and division of monomials & polynomials.
21. Equations as symbolic sentences:	The student will read an equation aloud and interpret it as a sentence.
22. Removing parentheses used to indicate multiplication:	The student will simplify equations by performing indicated multiplications in order to be able to remove parentheses. Minimum performance level will be correctly solving four problems from a set of five.
23. Adding and subtracting polynomials:	The student will demonstrate his ability to add & subtract polynomials by combining similar terms. An acceptable performance level will be 80% correct answers.
24. Inserting signs of grouping:	The student will clarify a set of problems by inserting signs of grouping.
25. Rules of Exponents:	The student will demonstrate an understanding of the rules of exponents by stating each and working an example of it. Minimum ability will be 80% accuracy.
26. The product of a binomial sum or difference:	The student will expand binomial sums or differences for ten problems. Minimum level of performance will be eight correct answers.

Arithmetic Operations with Algebraic Expressions

Concept	Behavioral Objective
27. Factoring the difference of two squares:	The student will accurately factor four of five problems involving a difference of two squares and will check by multiplying the factors.
28. Factoring out the common factors:	Four of five problems will be correctly solved when the student has factored out common factors.
29. Factoring polynomials:	The student will completely factor eight of ten expressions involving polynomials.
30. Multiplication by a common factor:	The student will demonstrate the ability to multiply by a common factor by correctly expanding four of five given expressions.
31. Special products:	The student will state the expansion formulas for products of a sum and difference, the square of a sum, cube of a sum and cube of a difference.
32. Extracting square roots:	The student will extract the square root of eight of ten given expressions.
33. Squaring binomials:	The student will demonstrate the ability to square binomials by correctly solving at least four of five representative problems.

Algebraic Fractions

Concept	Behavioral Objective
34. Properties of algebraic fractions:	The student will state the properties of algebraic fractions and will illustrate each with an example.

Algebraic Fractions

Concept	Behavioral Objective
35. Basic operations with algebraic fractions:	The student will correctly solve eight of ten equations involving the four basic operations with fractions.
36. The fractional exponents:	For a given set of radical expressions, the student will simplify by expressing in terms of fractional exponents with 80% accuracy.
37. Finding the least common denominator:	The student will express each denominator as a product of prime factors and will use these prime factors to find the LCD correctly for seven out of eight problems.
38. Reducing to lower terms:	The student will factor both numerator and denominator into prime factors and will cancel common factors to reduce four of five fractions to lowest terms.

Inequalities

Concept	Behavioral Objective
39. Properties of inequalities:	The student will state the properties which are common to both equalities and inequalities, and will give examples of properties which are different for inequalities.
40. Literal equation formulas:	The student will solve eight of ten representative equations involving variables.
41. Equivalent, identical and conditional equations:	From a list of ten equations, the student will correctly identify at least eight as eigher equivalent, identical, or conditional.

Inequalities

Concept	Behavioral Objective
42. Simplification of equations:	Given five equations, the student will simplify at least four.
43. Solution of equations in variables:	Given ten equations, the student will solve at least eight in terms of a given variable.
44. Analysis of a formula:	Given five formulas the student will analyze at least four into component parts.
45. Transposition of terms:	The student will define the term transposition.
46. Transposition of factors:	Given five equations, the student will demonstrate the transposition of factors with 80% accuracy.
47. Translating problems into equations:	Given five word problems, the student will translate at least four into equations.
48. Operations with dimensional units:	Given ten formulas, the student will analyze eight in Terms of dimensional units.
49. Solving systems of equations of two and three unknowns:	Given ten equations in terms of two and three unknowns, the student will solve at least eight algebraically.
49. Dimensional units and conversion:	Given five dimensional units, the student will convert at least four to other dimensional units.
50. Equations containing fractions and decimals:	Given ten equations containing fractions and decimals, the student will solve at least eight.

Inequalities

Concept	Behavioral Objective
51. Degree of an Equation:	Given five equations of various degrees, the student will identify at least four.
52. Solving linear equations:	Given ten linear equations, the student will solve at least four.
53. Algebraic solutions to variables:	Given five variable euqations, the student will solve at least four in terms of a specified variable.
54. Equations with unknowns in the denominators:	Given ten equations with variable fractions, the student will find the LCD for at least eight.
55. Proportions:	The student will give the definition for proportion.
56. Proportionality:	Given five proportionalities, the student will translate at least four in terms of equations.

Linear Graphs

Concept	Behavioral Objective
57. Coordinate notation:	The student will define coordinate notation in terms of a cortesian graph system.
58. Plotting points on graphs:	Given five ordered pair, the student will plot at least four.
59. Displaying data with graphs:	Given a set of data, the student will display the data by three graphs with 80% accuracy.

Linear Graphs

Concept	Behavioral Objective
60. Solving problems by means of graphs:	Given five problems, the student will solve at least four graphically.
61. Graphs of linear equations:	Given five linear equations, the student will graph at least four.
62. Determining linear function from two points:	Given ten sets of points, the student will determine the linear functions for at least eight.
63. Graphing parallel lines:	Given five equations representing parallel lines, the student will show that at least four are parallel.
64. Graphing perpendicular lines:	Given five linear equations, the student will find and graph at least four perpendicular lines.
65. Graphing lines parallel to the axis:	The student will state the linear equations parallel to the vertical and horizontal axis.
66. Graphing solutions of simultaneous equations:	Given five sets of linear equations, the student will find the solution for at least four sets.
67. The slope of a line and Y intersect:	The student will state the formula for the slope-intercept.
68. The distance formula:	Given five sets of points, the student will find the distance between at least four sets.
69. Graphic solution of system of linear equations:	The student will explain the Solution of systems of linear equations by the graphical method.

Linear Graphs

Concept	Behavioral Objective
70. Solution by addition and subtraction:	The student will explain solutions of linear equations by addition & subtraction.
71. Applications of simultaneous equations:	Given five problems of application, the student will demonstrate the solution of at least four by simultaneous equations.

Quadratic Equations

Concept	Behavioral Objective
72. Solution of $ax^2 + bx + c = 0$:	The student will solve the general quadratic equation $ax^2 + bx + c = 0$.
73. Solution by the quadratic formula:	The student will demonstrate the solution of at least four out of five quadratic equations by the use of the quadratic formula.

Exponential Notation

Concept	Behavioral Objective
74. The law of zero exponents:	The student will state the zero law for exponents.
75. The law of exponents in raising to powers:	The student will state the law for raising to a power to a power.
76. The law of exponents in extracting roots:	The student will state the law of exponents in extracting roots.

Exponential Notation

Concept	Behavioral Objective
77. The law of exponents based on 10:	The student will state the law of exponents based on 10.
78. Power variations:	The student will explain what is meant by power variation.
79. j-operator:	The student will demonstrate the use of the j-operator.
80. The one operator:	The student will demonstrate the one operator.
81. Complex numbers:	The student will identify complex numbers from a list of real complex numbers.
82. Complex equations:	The student will solve four out of five complex equations.
83. Squares of numbers:	The student will square five numbers with 80% accuracy.

Radicals

Concept	Behavioral Objective
84. Irrational numbers:	The student will define the term irrational number.
85. The square root of positive numbers:	Given five positive numbers, the student will find the square root of at least four.

Logarithmic Notation

Concept	Behavioral Objective
86. Definition of a logarithm:	The student will state the definition of a logarithm.

Concept	Behavioral Objective
87. Properties of logarithms:	The student will state the porperties of logarithms.
88. Characteristics:	The student will define the characteristics of a number for logarithm notations.
89. Mantissa:	The student will define mantissa of a number for logarithm notation.
90. Determining logs:	Given five numbers, the student will find their logs from a table with 80% accuracy.
91. Finding powers using logarithms:	Given five numbers, the student will find the powers using logarithms of base 10.
92. Finding roots using logarithms:	Given five numbers, the student will find specified roots of at least four by use of logarithms.
93. Interpolation using logarithms:	The student will explain interpolation by use of logarithms.
94. The logarithm of unity:	The student will explain why the logarithm of unity is zero.
95. Given log n, find n:	Given five problems of log n, the student will find n for at least four.
96. Given n, find log n:	Given five numbers the student will find log n of at least four.
97. Logarithms to the base e:	The student will define the value of e in logarithm notation.

Logarithmic Notation

Concept	Behavioral Objective
98. Time constant curves:	Given five representative problems, the student will find at least four time constant curves.

Sets and Logic

Concept	Behavioral Objective
99. Electric switch diagrams:	Given an electric switch diagram, the student will explain the diagram with 80% accuracy.
100. Venn diagrams:	Given three intersecting sets, the student will construct their Venn diagram with 80% accuracy.
101. Truth tables:	The student will explain the notation of truth tables with 80% accuracy.
102. Tautologies:	The student will explain tautologies with 80% accuracy.

Work and Power

Concept	Behavioral Objective
103. Work and power:	The student will define work and power.
104. Horsepower of a steam engine:	Given five representative problems, the student will find at least four horsepowers of steam engines.
105. Horsepower of a gas engine:	Given five representative problems, the student will find at least four horsepowers of gas engines.

Work and Power

Concept	Behavioral Objective
106. Brake horsepower:	The student will define brake horsepower.
107. Electrical powerwatt:	The student will define the electrical power watt.
108. Mechanical efficiency of machines:	The student will define what the mechanical efficiency of machines mean.

Speed Ratio and Pulley Gears

Concept	Behavioral Objective
109. Gear Trains:	Given a diagram of gear trains, the student will explain the diagram with 80% accuracy.
110. Idlers:	The student will explain how an idler works with 80% accuracy.
111. Finding the number of teeth for a given speed ratio:	Given a speed ratio, the student will explain how to find the number of gear teeth with 80% accuracy.
112. Compount gearing:	The student will explain compound gearing with 80% accuracy.
113. Worm and gear:	The student will explain the advantages of a worm and gear with 80% accuracy.
114. Trains and spur, bevel, and worm gearing:	The student will explain the advantages of trains, spur, bevel and worm gearing with 80% accuracy.

Quadratic Equations and Formulas

Concept		Behavioral Objective
115.	The meaning of equadratic equations:	The student will explain the meaning of a quadratic equations with 80% accuracy.

ADVANCED ALGEBRA

Exponents

Concept	Behavioral Objective
1. The meaning of powers whose exponents are either "1" or "0":	The student will state the meaning of exponents of 1, and 0 with 80% accuracy.
2. The meaning of powers with negative whole-number exponents:	The student will define the term negative exponents.
3. The reciprocals of powers and converting division to multiplication:	Given five division problems, the student will solve at least four by using the concept of reciprocal.
4. Converting multiplications to powers of divisions:	Given five examples, the student will convert multiplication to division by the use of powers for at least four.
5. Equivalent forms of fractions containing powers:	Given five examples of fractions containing powers, the student will convert at least four to equivalent forms.
6. Powers of fractions:	The student will work four out of five examples utilizing powers of fractions.
7. Powers of powers:	Given ten examples, the student will raise at least eight to a specific power.
8. Adding and subtracting letterterms which contain powers:	Given ten examples, the student will add and subtract letterterms which contain powers for at least eight.
9. A review of powers and the laws of exponents:	The student will give all the properties and laws for exponents with 80% accuracy.

Exponents

Concept	Behavioral Objective
10. The roots of multiplication and division:	The student will explain what is meant by the roots of multiplication and division.
11. The roots of powers:	Given five power problems, the student will find specified roots for at least four.
12. Simplifying radicals by factoring out perfect powers:	Given ten examples, the student will simplify specified radicals by factoring out perfect powers for at least eight.
13. Raising radicals to powers:	Given ten radicals, the student will raise at least eight to a specified power.
14. The meaning of powers with fraction exponents:	The student will explain the meaning of powers with fractional exponents with 80% accuracy.
15. Fractional exponents and the laws of exponents and radical powers and decimal exponents:	The student will state the laws related to fractional exponents and will correctly solve a problem involving each law.
16. The meaning of roots and radicals:	The student will state the definitions for the terms, and will find the roots to at least four out of five expressions involving radicals.

Linear Graphs and Slope

Concept	Behavioral Objective
17. Identifying linear equations in "x" and "y":	From a set of ten equations, the student will correctly identify at least eight according to whether they are linear or nonlinear.

Linear Graphs and Slope

Concept	Behavioral Objective
18. The intercepts of linear equations:	The student will identify the intercepts for a set of linear equations with a minimum performance level of 80% accuracy.
19. Graphing linear equations by plotting intercepts:	The student will use the intercepts to correctly graph the solutions to at least four of five given linear equations.
20. The slope of a line expressed as a signed number:	The student will express the slope in terms of signed numbers as determined from graphs of straight lines with at least 80% accuracy.
21. The slope of a line as a ratio of various pairs of chamges in "x" and "y":	The student will use the slope formula to determine the slope from coordinates of given points in at least four of five given cases.
22. The slope-intercept form of linear equations:	Given ten linear equations, the student will express at least eight in the slope intercept form.
23. Writing linear equations in slope intercept form:	Given the slope and y-intercept, the student will write the correct linear equation with at least 80% accuracy.
24. Determining the slope of a line through two known points:	The student will use the slope formula to determine the slope of a line through a pair of points in at least four out of five cases.
25. Slope and parallel lines:	The student will use slope to determine whether or not two lines are parallel. Minimum acceptable performance iwll be correct identification of four of five pairs of lines.

Linear Graphs and Slope

Concept	Behavioral Objective
26. The equations of horizontal and vertical lines:	The student will state the general formulas for vertical and horizontal lines; he will also identify what conditions exist for x and y in each case.

Common and Natural Logarithms

Concept	Behavioral Objective
27. The power-of-ten form of numbers greater than 1:	Given five numbers greater than one, the student will give the power-of-ten for at least four of the numbers.
28. Common logarithms of numbers greater than 1:	Given ten numbers greater than one, the student will give the logartthim for at least eight of the numbers.
29. Common logarithms of numbers between 0 and 1:	Given ten numbers between zero and one, the student will find the logarithm for at least eight of the numbers.
30. Logarithmic scales:	The student will give a definition for a log scale.
31. Semi-log and log-log graphs:	The student will construct graphs on semi-log and log-log paper.
32. Equations with formulas which contain "e":	The student will evaluate four out of five formulas containing the "e" function.
33. Natural (base "e") logarithms:	The student will give a definition for the base "e" logarithm.
34. Evaluating "Ln" formulas:	The student will evaluate eight of ten formulas containing "Ln" functions.
35. Evaluating "Ln" formulas when a conversion to power-of-"e" form is required:	The student will convert four out of five "Ln" function formulas to powers of "e" function formulas.

Logarithms and Exponentials: Laws and Formulas

Concepts	Behavioral Objective
36. The laws of common logarithms:	The student will state the three laws for common logarithms.
37. The laws of natural logarithms:	The student will state the three laws for natural logarithms.

TRIGONOMETRY

Right Triangles and Trigonometric Ratios

Concepts	Behavioral Objective
1. Relationship between θ and the coordinates of P:	Given a particular angle, the student will find the coordinates of the point P, on a unit circle.
2. Variations in right triangles problems:	The student will explain variations in right triangle problems.
3. Linear velocity:	The student will give a definition for linear velocity.
4. Right triangles and angle-sum principle:	The student will define a right triangle and state the angle-sum principle.
5. Right triangles and pythagorean theorem:	The student will state the pythagorean theorem and work four out of five problems using the pythagorean formula.
6. Scaling method and right triangles:	Given a right triangle, the student will construct five scale representations with 80% accuracy.
7. The definition of the tangent ratio:	The student will state the definition of the tangent ratio.
8. The tangent ratio of three specific angles:	Given the angles of 30°, 45° and 60°, the student will give the tangent ratios.
9. A partial table of tangent ratios:	Given specific angles, the student will construct a table with 80% accuracy.
10. The sine ratio--a partial table:	The student will define the sine ratio and construct a sine ratio table for three specified angles with 80% accuracy.

Right Triangles and Trigonometric Ratios

Concept	Behavioral Objective
11. The cosine ratio - a partial table:	The student will define the cosine ratio and construct a table.
12. The complete table of trig ratios:	The student will demonstrate his competency by finding eight out of ten trigonometric ratios from a table.
13. Finding unknown sides in right triangles:	The student will work four out of five problems involving unknown sides of a right triangle.
14. Finding unknown angles in right triangles:	The student will work four out of five problems involving unknown angles of right triangles.

Vectors and Phasors

Concept	Behavioral Objective
15. Definition of vectors and phasors:	The student will state the definition of a vector and a phasor.
16. Vectors on the rectangular coordinate system:	The student will construct four out of five vectors on a rectangular coordinate system.
17. The trig ratios of reference angles:	Given two reference angles, the student will give the trig ratios with 80% accuracy.
18. Finding the length and direction of a slanted vector:	Given a slanted vector, the student will find the length and direction.
19. Addition of vectors by the component method:	Given two vectors, the student will add them by the component method.

Vectors and Phasors

Concept	Behavioral Objective
20. Finding the length and direction of a resultant:	Given two vectors, the student will find the length and direction of the resultant vector.
21. Adding vectors which lie on the coordinate axes:	Given two vectors that lie on one of the coordinate axes, the student will find the resultant vector.
22. Finding the vector-addend:	Given a resultant vector and one addend, the student will find the other.
23. Adding more than two vectors:	Given three vectors, the student will find the resultant vector.
24. Vector opposites, the zero-vector, and the state of equilibrium:	The student will state the definition of vector opposites, zero vector and explain who they relate to the state of equilibrium.
25. Equilibriants and the state of equilibrium:	Given any two vectors, the student will find the equilibriant vector to produce a state of equilibrium.

Trigonometric Ratios of Standard-Position Angles

Concept	Behavioral Objective
26. Principle angles:	The student will state the definition of a principle angle.
27. Standard-position angles between 0° and 360°:	The student will convert any angle to standard-position angle.

Trigonometric Ratios of Standard-Position Angles

Concept	Behavioral Objective
28. Reference angles of standard-position angles between 0° and 360°:	The student will identify the reference angles for any angle between 0° and 360°.
29. Trigonometric ratios of angles between 90° and 360°:	Given any angle between 90° and 360°, the student will give all the trig. ratios with 80% accuracy.
30. Finding the functions of any angle:	Given any angle, the student will find all of the trig functions with 80% accuracy.
31. Finding standard-position angles corresponding to known ratios:	Given a known trig ratio, the student will find the standard angle.
32. Finding the angle given the function:	Given the value for any trig. function, the student will find the angle.
33. Trigonometric ratios of 0°, 90°, 180°, 270° and 360°:	The student will write all the trig ratios for 0°, 90°, 180°, 270° and 360°, with 80% accuracy.
34. The basic cycle of the sine-wave graph, graphing trig. functions:	The student will state the basic period for the sine-wave graph and sketch its graph.
35. Vector-addition and standard position angles:	The student will explain the relationship of vector addition to standard position angles.
36. Trigonometric ratios of angles greater than 360°:	The student will explain how to find the trig. ratios for any angle greater than 360°.
37. Other cycles of sine-wave graph:	Given different periods, the student will graph the sine-wave.

Trigonometric Ratios and Standard-Position Angles

Concept	Behavioral Objective
38. Sine and tangent of very small angles:	The student will state the approximation for sine and tangent of small angles.
39. Functions of angles in quadrants I, II, III, and IV:	Given angles in the I, II, III and IV quadrants, the student will find all the trig. functions with 80% accuracy.
40. Finding log of trig. functions:	Given five trig. functions, the student will find the logs with 80% accuracy.
41. Definition of a "degree" in angles measurement:	The student will give the definition of a degree of angular measurement.
42. Operations using minutes, degrees and seconds:	Given five angles, the student will convert minutes to degrees and minutes to seconds.
43. Definition of radian in angle measurement:	The student will state the definition of a radian.
44. The relationship between degrees and radians:	The student will convert degrees to radians.
45. Radians expressed in terms of "π":	Given five examples of radian measurement, the student will express the terms as multiples of π with 80% accuracy.
46. Trig. ratios of angles expressed in radians:	The student will specify a set of ratios of angles in terms of radians.

Oblique Triangles

Concept	Behavioral Objective
47. The definition of an ablique triangle:	The student will state the definition of an ablique triangle.

Oblique Triangles

Concept	Behavioral Objective
48. Solving oblique triangles by right-triangle methods:	Given five oblique triangles the student will solve them by right triangle methods with 80% accuracy.
49. The law of sines:	The student will state the law of sines.
50. Using the law of sines to find the unknown angles of oblique triangles:	Given an oblique triangle, the student will use the law of sine to find the unknown angles.
51. Limitations of using the law of sines:	The student will explain the limitations of the law of sines.
52. The law of cosines:	The student will state the law of cosines.
53. Using the law of cosines to fine unknown sides and angles of oblique triangles:	Given an oblique triangle the student will use the law of cosines to find the unknown sides and angles.
54. Right triangles-law of sines and law of cosines:	The student will explain how to utilize the law of sines and cosines to solve problems involving right triangles.

Applied Geometric Problems

Concept	Behavioral Objective
55. Central angles and arcs of circles:	The student will define the terms, central angle and arc of a circle.
56. Chords of circles:	The student will give an example of a chord of a circle.

Applied Geometric Problems

Concept	Behavioral Objective
57. Sectors and segments of circles:	The student will define the terms sector and segment of a circle.
58. Revolutions and rotational spees:	The student will define the terms revolutions and rotational speed.
59. Circumferential speed:	The student will define the term circumferential speed.
60. Angular measures of rotational speed:	The student will give an example of an angular measurement of rotational speed.
61. Circumferential speed and angular velocity (in radians):	The student will explain the relation between circumferential speed and angular velocity.
62. Areas of a triangle:	Given five triangles, the student will calculate the areas with 80% accuracy.

Identities and Interpolation

Concept	Behavioral Objective
63. Definitions of the cosecant, and secant, and cotangents of an angle:	The student will define the ratios, cosecant, secant and cotangents of an angle.
64. Finding the cosecant, secant, and cotangent of an angle:	Given five angles, the student will find the cosecant, secant and cotangent with 80% accuracy.
65. The ratio identities:	The student will state all of the ratio identities.
66. Pythagorean identities:	The student will state the Pythagorean identities.

Identities and Interpolation

Concept	Behavioral Objective
67. Simple identities:	The student will state the simple identities.
68. Reciprocal identities:	The student will state the reciprocal identities.
69. Verifying identities:	Given five identities, the student will verify them with 80% accuracy.
70. Similar right triangles:	Given five similar triangles, the student will show why they are similar with 80% accuracy.

Sine Wave Analysis

Concept	Behavioral Objective
71. Fundamental sine waves:	The student will graph an example of fundamental sine wave.
72. Non-fundamental sine waves:	The student will graphy an example of fundamental sine-wave.
73. Phase differences between non-fundamental and fundamental sine waves:	The student will explain the phase difference between non-fundamental and fundamental sine waves
74. The concepts of "lead" and "lag":	The student will explain the concepts of lead and lag in the phase difference of a sine wave.
75. Sine wave harmonics:	The student will explain sine wave harmonics.
76. Sketching fundamental sine waves:	Given five sine wave functions, the student will graph the sine waves with 80% accuracy.

Sine Wave Analysis

Concept	Behavioral Objective
77. Sketching non-fundamental sine waves:	The student will sketch the graph of a non-fundamental sine wave.
78. Sine waves with negative amplitudes:	The student will explain how a negative amplitude will affect the graph of a sine wave.

Complex Numbers

Concept	Behavioral Objective
79. j-operator:	The student will define the j-operator.
80. Real and imaginary numbers:	Given five examples the student will identify the real and imaginary numbers with 80% accuracy.
81. Complex numbers and their vector representation:	The student will represent a complex number on a coordinate graph system in terms of a vector.
82. Vector-addition by means of complex numbers:	Given five examples, the student will demonstrate vector addition by means of complex numbers with 80% accuracy.
83. Vector-Subtraction by means of complex numbers:	The student will explain vector subtraction by use of complex numbers.
84. The polar form of vectors which lies on an axis:	The student will convert five vectors to polar form with 80% accuracy.
85. Converting complex numbers from polar form to rectangular form:	Given five complex numbers in polar form, the student will convert at least four to the rectangular form.

Complex Numbers

Concept	Behavioral Objective
86. Graphs of complex numbers:	Given five complex numbers, the student will graph at least four on a complex plane.

Numerical Trigonometry (Applications)

Concept	Behavioral Objective
87. Measurement of angles:	The student will explain the various measurements of angles.
88. Mapping:	The student will define the term mapping.
89. Lattitudes and departures:	The student will explain the use of lattitude and departure.
90. Calculations of areas:	The student will solve five area calculation problems with 80% accuracy.
91. Measurements of distances in a vertical plane:	The student will explain how to do measurements in a vertical plane.

Periodic Functions

Concept	Behavioral Objective
92. Circular Functions:	The student will define circular function.
93. Graphing of $y=\sin\theta$:	The student will sketch the graph of $y=\sin\theta$.
94. Graph of $y=\cos\theta$:	The student will sketch the graph of $y=\cos\theta$.
95. Graphs of trig. functions:	The student will sketch all the graphs of the trig functions:

Periodic Functions

	Concept	Behavioral Objective
96.	Trig functions; $f(x) = \sin(x)$; $g(x) = \cos(x)$:	The student will explain the use of $f(x)$ in sketching trig. functions.
97.	Amplitude:	The student will define amplitude in terms of trig. functions.
98.	Frequency:	The student will define frequency in terms of trig. functions.
99.	Addition and sinusoidal waves:	Given five examples, the student will add sinusoidal waves with 80% accuracy.

The D. C. Circuit

	Concept	Behavioral Objective
100.	Difference of potential and electric current:	The student will define the terms of potential difference and electric current.
101.	E.M.F. and energy considerations:	The student will explain how E.M.F. and energy are related.
102.	Ohm's Law:	The student will state and explain the Ohm's Law.
103.	Voltage drops:	The student will explain the term voltage drop.
104.	Energy and power dissipated in a circuit:	The student will explain how how energy is dissipated in a circuit.
105.	Series and parallel circuits; equivalent resistance:	The student will draw diagrams indicating a parallel and a series circuit for a set of reisitors.
106.	Direction of current:	The student will explain conventional current flow and electronic current flow.

The D. C. Circuit

Concept	Behavioral Objective
107. Kirchhoff's rules:	The student will state Kirchhoff's circuit rules.
108. Battery E.M.F. and terminal voltage--internal resistance:	Given a representative example, the student will calculate the internal resistance and explain how it is related to E.M.F. and terminal voltage.
109. The wheatstone bridge:	The student will demonstrate the use of the wheatstone bridge.

Fundamental Elements of an A.C. Circuit

Concept	Behavioral Objective
110. Capacitance:	The student will define capacitance.
111. Capacitors in series:	The student will wire two capacitors in series.
112. Capacitors in parallel:	The student will wire two capacitors in parallel.
113. Energy requires to charge a capacitor:	The student will calculate the energy required to charge capacitors in series and in parallel.
114. Time for charging and discharging a capacitor; the timer constant:	The student will sketch the exponential curve representing the time for charging and the time for discharging a capacitor.
115. Inductance:	The student will define inductance.
116. Time constant of an L-R circuit:	The student will explain the time constant for a L-R circuit.

Fundamental Elements in an A.C. Circuit

	Concept	Behavioral Objective
117.	Phase relation in A.C. circuits:	The student will explain the phase relation in an A.C. circuit.
118.	The series A.C. circuits, phase diagram:	The student will draw a phase diagram for a series A.C. circuit.
119.	Reactance and impedance:	The student will define reactance and impedance.
120.	Resonance:	The student will define resonance.
121.	Power and power factor:	The student will explain the terms of power and power factors.

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